Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Make a recognisable sketch graph of $y = \|x-2\|$	B1	
		1	
1(b)	Find x-coordinate of intersection with $y = 3x - 4$	M1	
	Obtain $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
	Alternative method for question 1(b)		
	Solve the linear inequality $3x-4>2-x$, or corresponding equation	M1	
	Obtain critical value $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(x-2)^2 < (3x-4)^2$, or corresponding equation	M1	
	Obtain critical value $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
		3	

Question	Answer	Marks	Guidance
2	Use law of logarithm of a power and sum and remove logarithms	**M1**	
	Obtain a correct equation in any form, e.g. $3(2x+5)=(x+2)^2$	**A1**	
	Use correct method to solve a 3-term quadratic, obtaining at least one root	**M1**	
	Obtain final answer $x = 1+2\sqrt{3}$ or $1 + \sqrt{12}$ only	**A1**	
		4	

Question	Answer	Marks	Guidance
3(a)	Sketch the graph $y = \sec x$	**M1**	
	Sketch the graph $y=2-\frac{1}{2}x$, and justify the given statement	**A1**	
		2	
3(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	
3(c)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 0.88	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	**A1**	
		3	

Question	Answer	Marks	Guidance
4	Integrate by parts and reach $ax\tan x + b\int \tan x\,\mathrm{d}x$	**M1***	
	Obtain $x\tan x - \int \tan x\,\mathrm{d}x$	**A1**	
	Complete the integration, obtaining a term $\pm\ln\cos x$, or equivalent	**M1**	
	Obtain integral $x\tan x + \ln\cos x$, or equivalent	**A1**	
	Substitute limits correctly, having integrated twice	**DM1**	
	Use a law of logarithms	**M1**	
	Obtain answer $\frac{5}{18}\sqrt{3}\pi - \frac{1}{2}\ln 3$, or exact simplified equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
5(a)	Express LHS correctly as a single fraction	**B1**	
	Use $\cos(A \pm B)$ formula to simplify the numerator	**M1**	
	Use sin $2A$ formula to simplify the denominator	**M1**	
	Obtain the given result.	**A1**	
		4	

Question	Answer	Marks	Guidance
5(b)	Obtain an equation in $\tan 2x$ and use correct method to solve for x	**M1**	
	Obtain answer, e.g. 0.232	**A1**	
	Obtain second answer, e.g. 1.80	**A1**	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
6(a)	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term of the form $a\tan^{-1}(2y)$	**M1**	
	Obtain term $\frac{1}{2}\tan^{-1}(2y)$	**A1**	
	Obtain term $-e^{-x}$	**B1**	
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan^{-1}(by)$ and $ce^{\pm x}$	**M1**	
	Obtain correct answer in any form	**A1**	
	Obtain final answer $y = \frac{1}{2}\tan\left(2e^{-1} - 2e^{-x}\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(b)	State that y approaches $\frac{1}{2}\tan\left(2e^{-1}\right)$, or equivalent	B1FT	The FT is on correct work on a solution containing e^{-x}.
		1	

Question	Answer	Marks	Guidance
7(a)	State or imply $3y^2 + 6xy\frac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
	Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	Need to see $\frac{dy}{dx}$ factorised out prior to AG
	Obtain the given answer correctly	A1	AG
		4	
7(b)	Equate denominator to zero	*M1	
	Obtain $y = 2x$, or equivalent	A1	
	Obtain an equation in x or y	DM1	
	Obtain the point (1, 2)	A1	
	State the point $\left(\sqrt[3]{5}, 0\right)$	B1	Alternatively (1.71, 0).
		5	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j}$	B1	
	Use a correct method to find $\overrightarrow{MN}$	M1	
	Obtain $\overrightarrow{MN} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	A1	
		3	
8(b)	Use a correct method to form an equation for *MN*	M1	
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		2	
8(c)	Find $\overrightarrow{DP}$ for a point *P* on *MN* with parameter λ, e.g. $(2-\lambda, 1+2\lambda, -2+2\lambda)$	B1	
	Equate scalar product of $\overrightarrow{DP}$ and a direction vector for *MN* to zero and solve for λ	M1	
	Obtain $\lambda = \frac{4}{9}$	A1	
	State that the position vector of *P* is $\frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$	A1	
		4	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{1-2x}+\frac{C}{2+x}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = -2$, $B = 1$ and $C = 4$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
9(b)	Use correct method to find the first two terms of the expansion of $(1+2x)^{-1}$, $(1-2x)^{-1}$, $(2+x)^{-1}$ or $\left(1+\frac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1FT** **+ A1FT** **+ A1FT**	The FT is on *A*, *B* and *C*.
	Obtain final answer $1+5x-\frac{7}{2}x^2$	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	Solve for v or w	M1	
	Use $i^2 = -1$	M1	
	Obtain $v = -\dfrac{2i}{1+i}$ or $w = \dfrac{5+7i}{-1+i}$	A1	
	Multiply numerator and denominator by the conjugate of the denominator	M1	
	Obtain $v = -1 - i$	A1	
	Obtain $w = 1 - 6i$	A1	
		6	
10(b)(i)	Show a circle with centre 2 + 3i	B1	
	Show a circle with radius 1 and centre not at the origin	B1	
		2	
10(b)(ii)	Carry out a complete method for finding the least value of arg z	M1	
	Obtain answer $40.2°$ or 0.702 radians	A1	
		2	

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FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or power	**M1**	
	Obtain a correct equation free of logarithms, e.g. $3\left(x^3-3\right)=x^3$	**A1**	
	Obtain x = 1.65	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Substitute x = – 2, equate result to zero and obtain a correct equation, e.g. $-8a+20+8+b=0$	**B1**	
	Substitute x = – 1 and equate result to 2	**M1**	
	Obtain a correct equation, e.g. $-a+5+4+b=2$	**A1**	
	Solve for a or for b	**M1**	
	Obtain a = 3 and b = – 4	**A1**	
		5	

Question	Answer	Marks	Guidance
3	Use correct trig formulae to obtain an equation in tan x	***M1**	
	Using $\tan 45° = 1$, obtain a horizontal equation in $\tan x$ in any form	**DM1**	
	Reduce the equation to $\tan^2 x + \tan x - 1 = 0$, or 3-term equivalent	**A1**	
	Solve a 3-term quadratic in tan x, for x	**M1**	
	Obtain answer, e.g. $x = 31.7°$	**A1**	
	Obtain second answer, e.g. $x = 121.7°$, and no other in the interval	**A1**	Ignore answers outside the given interval.
		6	

Question	Answer	Marks	Guidance
4(a)	Separate variables correctly and attempt integration of at least one side	**M1**	
	Obtain term $\ln y$	**A1**	
	Obtain term of the form $\pm\ln(1-\cos x)$	**M1**	
	Obtain term $\ln(1-\cos x)$	**A1**	
	Use $x = \pi$, $y = 4$ to evaluate a constant, or as limits, in a solution containing terms of the form $a\ln y$ and $b\ln(1-\cos x)$	**M1**	
	Obtain final answer $y = 2(1-\cos x)$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
4(b)	Show a correct graph for $0 < x < 2\pi$ with the maximum at $x = \pi$	B1 FT	The FT is for graphs of the form $y = a(1 - \cos x)$, where a is positive.
		1	

Question	Answer	Marks	Guidance
5(a)	State $R = \sqrt{11}$	B1	
	Use trig formulae to find α	M1	
	Obtain $\alpha = 37.09°$	A1	
		3	
5(b)	Evaluate $\sin^{-1}\left(\frac{1}{\sqrt{11}}\right)$ to at least 2 dp (17.5484°)	B1 FT	The FT is on R.
	Use correct method to find a value of θ in the interval	M1	
	Obtain answer, e.g. 62.7°	A1	
	Use a correct method to obtain a second answer	M1	
	Obtain second answer, e.g. 170.2°, and no other in the interval	A1	Ignore answers outside the given interval.
		5	

Question	Answer	Marks	Guidance
6(a)	Carry out a relevant method to determine constants A and B such that $\dfrac{5a}{(2x-a)(3a-x)} = \dfrac{A}{2x-a} + \dfrac{B}{3a-x}$	**M1**	
	Obtain $A = 2$	**A1**	
	Obtain $B = 1$	**A1**	
		3	
6(b)	Integrate and obtain terms $\ln(2x-a) - \ln(3a-x)$	**B1 FT** **B1 FT**	The FT is on the values of A and B.
	Substitute limits correctly in a solution containing terms of the form $b\ln(2x-a)$ and $c\ln(3a-x)$, where $bc \neq 0$	**M1**	
	Obtain the given answer showing full and correct working	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Express general point of a line in component form, e.g. $(1 + 2s, 3 - s, 2 + 3s)$ or $(2 + t, 1 - t, 4 + 4t)$	**B1**	
	Equate at least two pairs of components and solve for s or for t	**M1**	
	Obtain correct answer for s or for t (possible answers are $-1, 6, \frac{2}{5}$ for s and $-3, 4, -\frac{1}{5}$ for t)	**A1**	
	Verify that all three component equations are not satisfied	**A1**	
	Show that the lines are not parallel and are thus skew	**A1**	
		5	
7(b)	Carry out correct process for evaluating the scalar product of the direction vectors	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer $19.1°$ or 0.333 radians	**A1**	
		3	

Question	Answer	Marks	Guidance
8(a)	Multiply numerator and denominator by 3 – i	**M1**	OE
	Obtain numerator – 10 + 10i or denominator 10	**A1**	
	Obtain final answer – 1 + i	**A1**	
		3	
8(b)	State or imply $r = \sqrt{2}$	**B1 FT**	
	State or imply that $\theta = \frac{3}{4}\pi$	**B1 FT**	
		2	
8(c)	State that *OA* and *BC* are parallel	**B1**	
	State that $BC = 2OA$	**B1**	
		2	

Question	Answer	Marks	Guidance
8(d)	Use angle $AOB = \arg u - \arg v = \arg\frac{u}{v}$	**M1**	
	Obtain the given answer	**A1**	
	Alternative method for question 8(d)		
	Obtain tan AOB from gradients of OA and OB and the $\tan(A \pm B)$ formula	**M1**	
	Obtain the given answer	**A1**	
	Alternative method for question 8(d)		
	Obtain $\cos AOB$ by using the cosine rule or a scalar product	**M1**	
	Obtain the given answer	**A1**	
		2	

Question	Answer	Marks	Guidance
9(a)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	
9(b)	Use the iterative formula $x_{n+1} = \dfrac{e^{2x_n} + 1}{e^{2x_n} - 1}$, or equivalent, correctly at least once	**M1**	
	Obtain final answer 1.20	**A1**	
	Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195,1.205)	**A1**	
		3	

Question	Answer	Marks	Guidance
9(c)	Use quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to – 8 and obtain a quadratic in e^{2x}	**M1**	
	Obtain $2\left(e^{2x}\right)^2 - 5e^{2x} + 2 = 0$	**A1**	OE
	Solve a 3-term quadratic in e^{2x} for x	**M1**	
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	**A1**	
	Alternative method for question 9(c)		
	Use quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to – 8, take square roots and obtain a quadratic in e^{x}	**M1**	
	Obtain $\sqrt{2}e^{2x} - e^{x} - \sqrt{2} = 0$	**A1**	OE
	Solve a 3-term quadratic in e^{x} for x	**M1**	
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	State or imply du = cos x dx	**B1**	
	Using double angle formula for sin2x and Pythagoras, express integral in terms of u and du.	**M1**	
	Obtain integral $\int 2\left(u-u^3\right)\mathrm{d}u$	**A1**	OE
	Use limits u = 0 and u = 1 in an integral of the form au^2+bu^4, where $ab\neq 0$	**M1**	$a+b$ or $a+b-0$ $\left(a=1 \text{ and } b=-\frac{1}{2}\right)$
	Obtain answer $\frac{1}{2}$	**A1**	
		5	
10(b)	Use product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and use a double angle formula	***M1**	
	Obtain an equation in one trig variable	**DM1**	
	Obtain $4\sin^2 x=1$, $4\cos^2 x=3$ or $3\tan^2 x=1$	**A1**	
	Obtain answer $x=\frac{1}{6}\pi$	**A1**	
		6	

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
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GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
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Mathematics-Specific Marking Principles	
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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	Use law of the logarithm of a product or power	**M1**
	Obtain a correct linear inequality in any form, e.g. $\ln 2 + (1 - 2x) \ln 3 < x \ln 5$	**A1**
	Solve for x	**M1**
	Obtain $x > \dfrac{\ln 6}{\ln 45}$	**A1**
		4

Question	Answer	Marks
2(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2 - 3x)^{-2}$ or $\left(1 - \dfrac{3}{2}x\right)^{-2}$	**M1**
	State correct first term $\dfrac{1}{4}$	**B1**
	Obtain the next two terms $\dfrac{3}{4}x + \dfrac{27}{16}x^2$	**A1 + A1**
		4
2(b)	State answer $\lvert x \rvert < \dfrac{2}{3}$, or equivalent	**B1**
		1

Question	Answer	Marks
3	Use tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**
	Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan θ in any correct form	**A1**
	Reduce the equation to $3\tan^2\theta + 4\tan\theta - 1 = 0$, or equivalent	**A1**
	Solve a 3-term quadratic for tan θ	**M1**
	Obtain a correct answer, e.g. 12.1°	**A1**
	Obtain a second correct answer, e.g. 122.9°, and no others in the given interval	**A1**
		6

Question	Answer	Marks
4(a)	Use product rule	**M1**
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	**A1**
	Equate derivative to zero and obtain an equation in one trigonometric ratio	**M1**
	Obtain $x = 1.43$ only	**A1**
		4
4(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	**M1**
	Show that it is a maximum point	**A1**
		2

Question	Answer	Marks
5(a)	Commence division and reach quotient of the form $2x + k$	**M1**
	Obtain quotient $2x - 1$	**A1**
	Obtain remainder 6	**A1**
		3
5(b)	Obtain terms $x^2 - x$ (**FT** on quotient of the form $2x + k$)	**B1FT**
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	**A1FT**
	Use $x = 1$ and $x = 3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	**M1**
	Obtain final answer $\frac{1}{\sqrt{3}}\pi + 6$, or exact equivalent	**A1**
		5

Question	Answer	Marks
6(a)	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1
	Use correct area formula and form an equation in r and x	M1
	Rearrange in the given form	A1
		3
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
6(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.35	A1
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	A1
		3

Question	Answer	Marks
7(a)	Use quotient or product rule	M1
	Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	A1
	Use Pythagoras to simplify the derivative	M1
	Justify the given statement	A1
		4

Question	Answer	Marks
7(b)	State integral of the form $a \ln(1+\sin x)$	*M1
	State correct integral $\ln(1+\sin x)$	A1
	Use limits correctly	DM1
	Obtain answer $\ln\frac{4}{3}$	A1
		4
8(a)	State $\frac{\mathrm{d}y}{\mathrm{d}x} = k\frac{y}{x\sqrt{x}}$, or equivalent	B1
	Separate variables correctly and attempt integration of at least one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $-2k\frac{1}{\sqrt{x}}$, or equivalent	A1
	Use given coordinates to find k or a constant of integration c in a solution containing terms of the form $a \ln y$ and $\frac{b}{\sqrt{x}}$, where $ab \neq 0$	M1
	Obtain $k = 1$ and $c = 2$	A1 + A1
	Obtain final answer $y = \exp\left(-\frac{2}{\sqrt{x}} + 2\right)$, or equivalent	A1
		8

Question	Answer	Marks
8(b)	State that y approaches e^2 (FT *their* c in part **(a)** of the correct form)	**B1FT**
		1
9(a)	State $\overrightarrow{AB}\left(\text{or } \overrightarrow{BA}\right)$ and $\overrightarrow{BC}\left(\text{or } \overrightarrow{CB}\right)$ in vector form	**B1**
	Calculate their scalar product	**M1**
	Show product is zero and confirm angle *ABC* is a right angle	**A1**
		3
9(b)	Use correct method to calculate the lengths of *AB* and *BC*	**M1**
	Show that $AB = BC$ and the triangle is isosceles	**A1**
		2
9(c)	State a correct equation for the line through *B* and *C*, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(-2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$	**B1**
	Taking a general point of *BC* to be *P*, form an equation in λ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle *OBP* (or *OCP*), *or* setting the derivative of $\left\|\overrightarrow{OP}\right\|$ to zero	**M1**
	Solve and obtain $\lambda = -\frac{5}{9}$	**A1**
	Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent	**A1**

Question	Answer	Marks
	Alternative method for question 9(c)	
	Use a scalar product to find the projection *CN* (or *BN*) of *OC* (or *OB*) on *BC*	**M1**
	Obtain answer $CN = \frac{5}{3}\left(\text{or } BN = \frac{14}{3}\right)$	**A1**
	Use Pythagoras to find *ON*	**M1**
	Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent	**A1**
		4

Question	Answer	Marks
10(a)(i)	Multiply numerator and denominator by $a - 2\text{i}$, or equivalent	**M1**
	Use $\text{i}^2 = -1$ at least once	**A1**
	Obtain answer $\frac{6}{a^2+4} + \frac{3ai}{a^2+4}$	**A1**
		3
10(a)(ii)	Either state that $\arg u = -\frac{1}{3}\pi$ or express u^* in terms of *a* (FT on *u*)	**B1**
	Use correct method to form an equation in *a*	**M1**
	Obtain answer $a = -2\sqrt{3}$	**A1**
		3

Question	Answer	Marks
10(b)(i)	Show the perpendicular bisector of points representing 2i and 1 + i	**B1**
	Show the point representing 2 + i	**B1**
	Show a circle with radius 2 and centre 2 + i (FT on the position of the point for 2 + i)	**B1FT**
	Shade the correct region	**B1**
		4
10(b)(ii)	State or imply the critical point 2 + 3i	**B1**
	Obtain answer 56.3° or 0.983 radians	**B1**
		2

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

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AWRT	Answer Which Rounds To

Question	Answer	Marks
1	Commence division and reach partial quotient $3x^2 + kx$	**M1**
	Obtain quotient $3x^2 + 2x - 1$	**A1**
	Obtain remainder $2x - 5$	**A1**
		3

Question	Answer	Marks
2	State or imply $2 \ln y = \ln A + kx$	**B1**
	Substitute values of $\ln y$ and x, or equate gradient of line to k, and solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for $\ln A$	**M1**
	Obtain $A = 3.31$	**A1**
	Alternative method for question 2	
	Obtain two correct equations in y and x by substituting y- and x- values in the given equation	**B1**
	Solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for A	**M1**
	Obtain $A = 3.31$	**A1**
		5

Question	Answer	Marks
3	Commence integration and reach $ax^{\frac{5}{2}}\ln x + b\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	M1*
	Obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{2}{5}\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	A1
	Complete the integration and obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{4}{25}x^{\frac{5}{2}}$, or equivalent	A1
	Use limits correctly, having integrated twice e.g $\frac{2}{5}\times 32\ln 4 - \frac{4}{25}\times 32 - \left(\frac{2}{5}\times 0\right) + \frac{4}{25}$	DM1
	Obtain answer $\frac{128}{5}\ln 2 - \frac{124}{25}$, or exact equivalent	A1
		5

Question	Answer	Marks
4	Use correct product rule	M1
	Obtain correct derivative in any form, e.g. $-\sin x \sin 2x + 2\cos x \cos 2x$	A1
	Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$	M1
	Equate derivative to zero and obtain an equation in one trig function	M1
	Obtain $3\sin 2x = 1$, or $3\cos 2x = 2$ or $2\tan 2x = 1$	A1
	Solve and obtain $x = 0.615$	A1
		6

Question	Answer	Marks
5(a)	State $R = \sqrt{7}$	B1
	Use trig formulae to find α	M1
	Obtain $\alpha = 57.688°$	A1
		3
5(b)	Evaluate $\cos-1\left(\frac{1}{\sqrt{7}}\right)$ to at least 3 d.p. (67.792°) (**FT** is on *their R*)	B1 FT
	Use correct method to find a value of θ in the interval	M1
	Obtain answer, e.g. 5.1°	A1
	Obtain second answer, e.g.117.3°, only	A1
		4

6(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form e.g. $\frac{(1+3x^4)-x\times 12x^3}{(1+3x^4)^2}$	A1
	Equate derivative to zero and solve for x	M1
	Obtain answer 0.577	A1
		4

Question	Answer	Marks
6(b)	State or imply $\mathrm{d}u = 2\sqrt{3}x\,\mathrm{d}x$, or equivalent	**B1**
	Substitute for x and $\mathrm{d}x$	**M1**
	Obtain integrand $\dfrac{1}{2\sqrt{3}\left(1+u^2\right)}$, or equivalent	**A1**
	State integral of the form $a\tan^{-1}u$ and use limits $u = 0$ and $u = \sqrt{3}$ (or $x = 0$ and $x = 1$) correctly	**M1**
	Obtain answer $\dfrac{\sqrt{3}}{18}\pi$, or exact equivalent	**A1**
		5

Question	Answer	Marks
7	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln(y-1)$	**B1**
	Carry out a relevant method to determine A and B such that $\frac{1}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$	**M1**
	Obtain $A = \frac{1}{2}$ and $B = -\frac{1}{2}$	**A1**
	Integrate and obtain terms $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$ $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$, or equivalent (**FT** is on A and B)	**A1 FT** **+ A1 FT**
	Use $x = 0$, $y = 2$ to evaluate a constant, or as limits in a solution containing terms of the form $a\ln(y-1)$, $b\ln(x+1)$ and $c\ln(x+3)$, where $abc \neq 0$	**M1**
	Obtain correct answer in any form	**A1**
	Obtain final answer $y = 1 + \sqrt{\left(\frac{3x+3}{x+3}\right)}$, or equivalent	**A1**
		9

Question	Answer	Marks
8(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $x - y = 3$ and $3x + y = 5$	**A1**
	Solve and obtain answer $z = 2 - i$	**A1**
		4
8(b)(i)	Show a point representing $2 + 2i$	**B1**
	Show a circle with radius 1 and centre not at the origin (**FT** is on the point representing the centre)	**B1 FT**
	Show the correct half line from $4i$	**B1**
	Shade the correct region	**B1**
		4
8(b)(ii)	Carry out a complete method for finding the least value of $\text{Im}\, z$	**M1**
	Obtain answer $2 - \frac{1}{2}\sqrt{2}$, or exact equivalent	**A1**
		2

Question	Answer	Marks
9(a)	State $\cos p = \dfrac{k}{1+p}$	B1
	Differentiate both equations and equate derivatives at $x = p$	M1
	Obtain a correct equation in any form, e.g. $-\sin p = -\dfrac{k}{(1+p)^2}$	A1
	Eliminate k	M1
	Obtain the **given answer** showing sufficient working	A1
		5
9(b)	Use the iterative formula correctly at least once	M1
	Obtain final answer $p = 0.568$	A1
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	A1
		3
9(c)	Use a correct method to find k	M1
	Obtain answer $k = 1.32$	A1
		2

Question	Answer	Marks
10(a)	State that the position vector of M is 3**i** + **j**	B1
	Use a correct method to find the position vector of N	M1
	Obtain answer $\frac{10}{3}\mathbf{i}+2\mathbf{j}+2\mathbf{k}$	A1
	Use a correct method to form an equation for MN	M1
	Obtain correct answer in any form, e.g. $\mathbf{r}=3\mathbf{i}+\mathbf{j}+\lambda\left(\frac{1}{3}\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$	A1
		5
10(b)	State or imply $\mathbf{r}=\mu(2\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ as equation for OB	B1
	Equate sufficient components of MN and OB and solve for λ or for μ	M1
	Obtain $\lambda=3$ or $\mu=2$ and position vector $4\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ for P	A1
		3
10(c)	Carry out correct process for evaluating the scalar product of direction vectors for OP and MP, or equivalent	M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1
	Obtain answer 21.6°	A1
		3

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

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GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x = -7$ and $x = -1$	**A1**
	State final answer $-7 < x < -1$	**A1**
	Alternative method for question 1	
	Obtain critical value $x = -1$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
	Obtain critical value $x = -7$ similarly	**B2**
	State final answer $-7 < x < -1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**
		4

Question	Answer	Marks
2	Commence integration and reach $a(2-x)e^{-2x}+b\int e^{-2x}\,\mathrm{d}x$, or equivalent	M1*
	Obtain $-\frac{1}{2}(2-x)e^{-2x}-\frac{1}{2}\int e^{-2x}\,\mathrm{d}x$, or equivalent	A1
	Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x}+\frac{1}{4}e^{-2x}$, or equivalent	A1
	Use limits correctly, having integrated twice	DM1
	Obtain answer $\frac{1}{4}(3-e^{-2})$, or exact equivalent	A1
		5

Question	Answer	Marks
3(a)	Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent	**B1**
	Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent	**B1**
		2
3(b)	Solve a 3-term quadratic for u	**M1**
	Obtain root $\frac{1}{2}\left(-1+\sqrt{5}\right)$, or decimal in [0.61, 0.62]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer $x = -0.481$ only	**A1**
		4

Question	Answer	Marks
4(a)	Use the product rule	**M1**
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	**M1**
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent	**A1**
		3
4(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	**B1**
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$	**M1**
	Obtain answer $p = -1$	**A1**
		3

Question	Answer	Marks
5	Use tan 2A formula to express RHS in terms of tan θ	**M1**
	Use tan ($A \pm B$) formula to express LHS in terms of tan θ	**M1**
	Using tan 45° = 1, obtain a correct horizontal equation in any form	**A1**
	Reduce equation to $2\tan^2\theta + \tan\theta - 1 = 0$	**A1**
	Solve a 3-term quadratic and find a value of θ	**M1**
	Obtain answer $\theta = 26.6°$ and no other	**A1**
		6

Question	Answer	Marks
6(a)	Sketch a relevant graph, e.g. $y = x^5$	B1
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	B1
		2
6(b)	State a suitable equation, e.g. $x = \dfrac{4x^5 + 2}{5x^4 - 1}$	B1
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	B1
		2
6(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.267	A1
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	A1
		3

Question	Answer	Marks
7(a)	State or imply the form $\frac{A}{2x-1}+\frac{B}{2x+1}$ and use a relevant method to find A or B	**M1**
	Obtain $A = 1$, $B = -1$	**A1**
		2
7(b)	Square the result of part (**a**) and substitute the fractions of part (**a**)	**M1**
	Obtain the given answer correctly	**A1**
		2
7(c)	Integrate and obtain $-\frac{1}{2(2x-1)}-\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(2x+1)-\frac{1}{2(2x+1)}$, or equivalent	**B3, 2, 1, 0**
	Substitute limits correctly	**M1**
	Obtain the given answer correctly	**A1**
		5

Question	Answer	Marks
8(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of C	**M1**
	Obtain answer $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, or equivalent	**A1**
	Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and AD	**M1**
	Show that $ABCD$ has a pair of unequal adjacent sides	**A1**
	Alternative method for question 8(a)	
	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of C	**M1**
	Obtain answer $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, or equivalent	**A1**
	Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent	**M1**
	Show that the diagonals of $ABCD$ are not perpendicular	**A1**
		5
8(b)	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer 100.3°	**A1**
		3

Question	Answer	Marks
8(c)	Use a correct method to calculate the area, e.g. calculate *AB.AC* sin *BAD*	**M1**
	Obtain answer 11.0 (**FT** on angle BAD)	**A1 FT**
		2

Question	Answer	Marks
9(a)	Eliminate *u* or *w* and obtain an equation *w* or *u*	**M1**
	Obtain a quadratic in *u* or *w*, e.g. $u^2 - 2iu - 6 = 0$ or $w^2 + 2iw - 6 = 0$	**A1**
	Solve a 3-term quadratic for *u* or for *w*	**M1**
	Obtain answer $u = \sqrt{5} + i$, $w = \sqrt{5} - i$	**A1**
	Obtain answer $u = -\sqrt{5} + i$, $w = -\sqrt{5} - i$	**A1**
		5
9(b)	Show the point representing $2 + 2i$	**B1**
	Show a circle with centre $2 + 2i$ and radius 2 (**FT** is on the position of $2 + 2i$)	**B1 FT**
	Show half-line from origin at 45° to the positive *x*-axis	**B1**
	Show line for Re $z = 3$	**B1**
	Shade the correct region	**B1**
		5

Question	Answer	Marks
10(a)	State or imply $\dfrac{dV}{dt} = -k\sqrt{h}$	**B1**
	State or imply $\dfrac{dV}{dh} = 2\pi rh - \pi h^2$, or equivalent	**B1**
	Use $\dfrac{dV}{dt} = \dfrac{dV}{dh}.\dfrac{dh}{dt}$	**M1**
	Obtain the **given answer** correctly	**A1**
		4
10(b)	Separate variables and attempt integration of at least one side	**M1**
	Obtain terms $\dfrac{4}{3}rh^{\frac{3}{2}} - \dfrac{2}{5}h^{\frac{5}{2}}$ and $-Bt$	**A3, 2, 1, 0**
	Use $t = 0$, $h = r$ to find a constant of integration c	**M1**
	Use $t = 14$, $h = 0$ to find B	**M1**
	Obtain correct c and B, e.g. $c = \dfrac{14}{15}r^{\frac{5}{2}}$, $B = \dfrac{1}{15}r^{\frac{5}{2}}$	**A1**
	Obtain final answer $t = 14 - 20\left(\dfrac{h}{r}\right)^{\frac{3}{2}} + 6\left(\dfrac{h}{r}\right)^{\frac{5}{2}}$, or equivalent	**A1**
		8

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $2^2(3x-1)^2 < (x+1)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Form and solve a 3-term quadratic, or solve two linear equations for x	**M1**	e.g. $35x^2 - 26x + 3 = 0$
	Obtain critical values $x = \frac{3}{5}$ and $x = \frac{1}{7}$	**A1**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**A1**	Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone $\leqslant$ for $<$ in the final answer. Fractions need not be in lowest terms.
	Alternative method for Question 1		
	Obtain critical value $x = \frac{3}{5}$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = \frac{1}{7}$ similarly	**B2**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**B1**	OE. Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone $\leqslant$ for $<$ in the final answer. Fractions need not be in lowest terms.
		4	

Question	Answer	Marks	Guidance
2	Reduce to a 3-term quadratic $u^2+6u-1=0$ OE	B1	Allow '= 0' implied
	Solve a 3-term quadratic for u	M1	
	Obtain root $\sqrt{10}-3$	A1	
	Obtain answer $x = -1.818$ only	A1	The question asks for 3 d.p.
	Reject $-\sqrt{10}-3$ correctly	B1	e.g. by stating that $e^x>0$ or $\ln\left(-10-\sqrt{3}\right)$ is impossible Not "math error".
	Alternative method for Question 2		
	Rearrange to obtain a correct iterative formula	B1	e.g. $x_{n+1}=-\ln\left(6+e^{x_n}\right)$
	Use the iterative process at least twice	M1	
	Obtain answer $x = -1.818$	A1	
	Show sufficient iterations to at least 4 d.p. to justify $x = -1.818$	A1	$1, -2.165..., -1.811..., -1.819..., -1.818..., -1.818...$
	Clear explanation of why there is only one real root	B1	
		5	

Question	Answer	Marks	Guidance
3(a)	Use correct trig expansions and obtain an equation in sin x and cos x	*M1	
	Use correct exact trig ratios for $30°$ in *their* expansion	B1 FT	e.g. $\cos x\left(\frac{\sqrt{3}}{2}-1\right)=\sin x\left(\sqrt{3}-\frac{1}{2}\right)$
	Obtain an equation in tan x	DM1	Allow if their error in line 1 was a sign error
	Obtain $\tan x=\dfrac{2-\sqrt{3}}{1-2\sqrt{3}}$ from correct working	A1	AG
		4	
3(b)	Obtain answer in the given interval, e.g.173.8°	B1	Accept $174°$, $354°$ or better
	Obtain a second answer and no other in the given interval, e.g. 353.8°	B1	Ignore answers outside the given interval. Treat answers in radians (3.03 and 6.17) as a misread.
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct double angle formula or t-substitution twice	M1	
	Obtain $\dfrac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$ from correct working	A1	AG
		2	

Question	Answer	Marks	Guidance
4(b)	Express $\tan^2\theta$ in terms of $\sec^2\theta$	M1	$\left(\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sec^2\theta \pm 1\right)\mathrm{d}\theta\right)$
	Integrate and obtain terms $\tan\theta - \theta$	A1	Accept with a mixture of x and θ
	Substitute limits correctly in an integral of the form $a\tan\theta + b\theta$, where $ab \neq 0$	M1	$\left(\sqrt{3} - \frac{\pi}{3} - \frac{1}{\sqrt{3}} + \frac{\pi}{6}\right)$ Allow if trig. not substituted
	Obtain answer $\frac{2}{3}\sqrt{3} - \frac{1}{6}\pi$	A1	or equivalent exact 2-term expression
		4	

Question	Answer	Marks	Guidance
5(a)	Use quadratic formula and $\mathrm{i}^2 = -1$	M1	
	Obtain answers $p\mathrm{i} + \sqrt{q - p^2}$ and $p\mathrm{i} - \sqrt{q - p^2}$	A1	Accept $\frac{2p\mathrm{i} \pm \sqrt{-4p^2 + 4q}}{2}$ and ISW
		2	
5(b)	State or imply that the discriminant must be negative	M1	
	State condition $q < p^2$	A1	
		2	

Question	Answer	Marks	Guidance
5(c)	Carry out a correct method for finding a relation, e.g. use the fact that the argument of one of the roots is $(\pm)60°$	M1	
	State a correct relation in any form, e.g. $\frac{p}{\sqrt{q-p^2}} = (\pm)\sqrt{3}$	A1	
	Simplify to $q = \frac{4}{3}p^2$	A1	
	Alternative method for Question 5(c)		
	Carry out a correct method for finding a relation, e.g. use the fact that the sides have equal length	M1	
	State a correct relation in any form, e.g. $4\left(q-p^2\right) = p^2 + q - p^2$	A1	
	Simplify to $q = \frac{4}{3}p^2$	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	Use correct chain rule **or** correct quotient rule to differentiate x or y	**M1**	
	Obtain $\frac{dx}{dt} = \frac{3}{2+3t}$ **or** $\frac{dy}{dt} = \frac{2}{(2+3t)^2}$	**A1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain answer $\frac{2}{3(2+3t)}$	**A1**	OE. Express as a simple fraction but not necessarily fully cancelled.
	Explain why this is always positive	**A1**	For correct gradient. e.g. x is only defined for $2+3t>0$ hence gradient > 0
	Alternative method for Question 6(a)		
	Form equation in x and y only	**M1**	
	Obtain $y = \frac{e^x - 2}{3e^x}\left(= \frac{1}{3} - \frac{2}{3}e^{-x}\right)$	**A1**	OE
	Differentiate	**M1**	
	Obtain $y' = \frac{2}{3}e^{-x}$	**A1**	OE
	Explain why this is always positive	**A1**	
		5	

Question	Answer	Marks	Guidance
6(b)	Obtain $y = -\frac{1}{3}$ when $x = 0$	**B1**	
	Use a correct method to form the given tangent	**M1**	$\left(\frac{y+\frac{1}{3}}{x} = \frac{2}{3}\right)$
	Obtain answer $3y = 2x - 1$	**A1**	OE
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct quotient rule or correct product rule	**M1**	e.g. $\frac{dy}{dx} = \frac{\sqrt{x}.\frac{1}{1+x^2} - \tan^{-1}x.\frac{1}{2\sqrt{x}}}{x}$
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and remove inverse tangent	**M1**	
	Obtain $a = \tan\left(\frac{2a}{1+a^2}\right)$ from correct working	**A1**	AG. Accept with x in place of a.
		4	

Question	Answer	Marks	Guidance
7(b)	Calculate the value of a relevant expression or pair of expressions at $a = 1.3$ and $a = 1.5$	M1	Must be using radians
	Complete the argument correctly with correct calculated values	A1	e.g. $1.3 < 1.448, \quad 1.5 > 1.322 \ \left(0.148, -0.178\right)$
		2	
7(c)	Use the iterative process $a_{n+1} = \tan\left(\dfrac{2a_n}{1+a_n{}^2}\right)$ correctly at least twice	M1	
	Obtain final answer 1.39	A1	
	Show sufficient iterations to at least 4 d.p. to justify 1.39 to 2 d.p. or show there is a sign change in the interval (1.385, 1.395)	A1	Allow recovery
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$	B1	OE. Allow ±
	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. their $\overrightarrow{AB}$ and a direction vector for l	M1	$\left(2+2-3=1\right)$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	M1	$\cos^{-1}\left(\dfrac{1}{\sqrt{6}\sqrt{14}}\right)$
	Obtain answer $83.7°$ or 1.46 radians	A1	Or answers rounding to $83.7°$ or 1.46 radians
		4	

Question	Answer	Marks	Guidance
8(b)	State or imply $\pm\overrightarrow{AP}$ and $\pm\overrightarrow{BP}$ in component form, i.e. $(1 + \lambda, 1 - 2\lambda, \lambda)$ and $(-1 + \lambda, 2 - 2\lambda, 3 + \lambda)$, or equivalent	**B1**	
	Form an equation in λ by equating moduli or by using $\cos BAP = \cos ABP$	***M1**	
	Obtain a correct equation in any form $(1+\lambda)^2+(1-2\lambda)^2+\lambda^2=(\lambda-1)^2+(2-2\lambda)^2+(\lambda+3)^2$	**A1**	Or $(1+\lambda)\sqrt{14-4\lambda+6\lambda^2}=(13-\lambda)\sqrt{2-2\lambda+6\lambda^2}$ $\left(83\lambda^3-528\lambda^2+207\lambda-162=0\right)$
	Solve for λ and obtain position vector	**DM1**	[$\lambda = 6$]
	Obtain correct position vector for P in any form, e.g. $(8, -9, 7)$ or $8\mathbf{i} - 9\mathbf{j} + 7\mathbf{k}$	**A1**	Accept coordinates
		5	

Question	Answer	Marks	Guidance
9(a)	Use correct product rule or correct quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	$y'=\dfrac{x^{-\frac{2}{3}}}{x}-\dfrac{2}{3}x^{-\frac{5}{3}}\ln x$
	Equate 2 term derivative to zero and solve for x	**M1**	
	Obtain answer $x=\mathrm{e}^{\frac{3}{2}}$	**A1**	Or exact equivalent
	Obtain answer $y=\dfrac{3}{2\mathrm{e}}$	**A1**	Or exact equivalent
		5	

Question	Answer	Marks	Guidance
9(b)	Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\,dx$	*M1	
	Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\,dx$	A1	
	Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$	A1	OE
	Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$	DM1	$6\ln 8 - 9\times 2 - 0 + 9$
	Obtain $18\ln 2 - 9$ from full and correct working	A1	AG need to see $\ln 8 = 3\ln 2$
		5	

Question	Answer	Marks	Guidance
10	State a suitable form of partial fractions for $\frac{1}{x^2(1+2x)}$	**B1**	e.g. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{1+2x}$ or $\frac{Ax+B}{x^2} + \frac{C}{1+2x}$
	Use a relevant method to determine a constant	**M1**	
	Obtain one of $A = -2$, $B = 1$ and $C = 4$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	Separate variables correctly and integrate at least one term	**M1**	
	Obtain terms $-2\ln x - \frac{1}{x} + 2\ln(1+2x)$ and t	**B3 FT**	The FT is on A, B and C. Withhold B1 for each error or omission.
	Evaluate a constant, or use limits $x = 1$, $t = 0$ in a solution containing terms t, $a\ln x$ and $b\ln(1+2x)$, where $ab \neq 0$	**M1**	
	Obtain a correct expression for t in any form, e.g. $t = -\frac{1}{x} + 2\ln\left(\frac{1+2x}{3x}\right) + 1$	**A1**	
		11	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

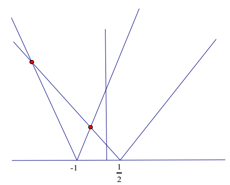
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

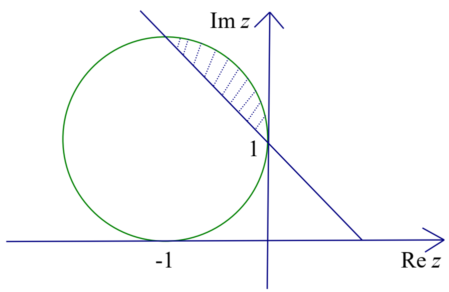
FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(2x-1)^2 < 3^2(x+1)^2$, or corresponding quadratic equation	**B1**	e.g. $5x^2+22x+8=0$ Allow recovery from 'invisible brackets' on RHS
	Form and solve a 3-term quadratic in x	**M1**	
	Obtain critical values $x = -4$ and $x = -\frac{2}{5}$	**A1**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**A1**	Do not condone ⩽ for <, or ⩾ for > in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty, -4) \cup (-0.4, \infty)$
	Alternative method for Question 1		
	Obtain critical value $x = -4$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{2}{5}$ similarly	**B2**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**B1**	Do not condone ⩽ for <, or ⩾ for > in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty, -4) \cup (-0.4, \infty)$
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre – 1 + i.	**B1**	Need some indication of scale or a correct label. Could just be mark(s) on the axes
	Show a circle with radius 1 and centre not at the origin (or relevant part thereof).	**B1**	
	Show correct half line from 1(or relevant part thereof) .	**B1**	
	Shade the correct region on a correct diagram.	**B1**	
		**4**	N.B. If they have very different scales on *their* 2 axes the diagram must match *their* scale - the 'circle' should be an ellipse. Allow freehand diagrams with clear correct intention.

Question	Answer	Marks	Guidance
3(a)	State or imply $\ln x = \ln A - y\ln 3$	**B1**	$\left(y = -\frac{1}{\ln 3}\ln x + \frac{\ln A}{\ln 3}\right)$
	State that the graph of y against $\ln x$ has an equation that is *linear* in y and $\ln x$, or has an equation of the standard form '$y = mx + c$' and is thus a straight line	**B1**	Must be a correct statement. Accept if the 2 equations are written side by side with no comment. An equation with $y\ln 3$ should be compared with the form $py + q\ln x = c$.
	State that the gradient is $-\frac{1}{\ln 3}$	**B1**	OE. Exact answer required. ISW after a correct statement.
		3	

Question	Answer	Marks	Guidance
3(b)	Substitute ln x = 0, y = 1.3 and use correct method to solve for A	**M1**	$(\ln A = 1.3\ln 3)$ Follow *their* equation in y and $\ln x$. Must be substituting ln$x$ = 0, not x = 0. ln 0 'used' in the solution scores M0A0.
	Obtain answer A = 4.17 only	**A1**	Must be 2 d.p. as specified in question
		2	

Question	Answer	Marks	Guidance
4	Commence integration and reach $ax\tan^{-1}\frac{1}{2}x + b\int x\frac{1}{c+x^2}\mathrm{d}x$	***M1**	OE. Denominator might be $1+\frac{x^2}{4}$ or $2+\frac{x^2}{2}$.
	Obtain $x\tan^{-1}\left(\frac{1}{2}x\right) - \int x.\ \frac{2}{4+x^2}\ \mathrm{d}x$	**A1**	OE
	Complete integration and obtain $x\tan^{-1}\left(\frac{1}{2}x\right) - \ln\left(4+x^2\right)$	**A1**	OE e.g. with $\ln\left(1+\frac{x^2}{4}\right)$
	Substitute limits correctly in an expression of the form $px\tan^{-1}x + q\ln\left(c+x^2\right)$	**DM1**	$2\tan^{-1}1 - \ln 8 + \ln 4$ OE
	Obtain final answer $\frac{1}{2}\pi - \ln 2$	**A1**	OE exact answer. Needs a value for $\tan^{-1}1$ and a single log term
	Alternative method for Question 4		
	Use the substitution $\theta = \tan^{-1}\frac{x}{2}$ to obtain $\lambda\int 2\theta\sec^2\theta\,\mathrm{d}\theta$ and reach $p\theta\tan\theta + q\int\tan\theta\,\mathrm{d}\theta$	***M1**	

Question	Answer	Marks	Guidance
4	Obtain $2\theta\tan\theta - 2\int \tan\theta \, d\theta$	**A1**	OE
	Complete integration and obtain $2\theta\tan\theta + 2\ln(\cos\theta)$	**A1**	OE
	Substitute correct limits correctly in an expression of the form $r\theta\tan\theta + s\ln(\cos\theta)$	**DM1**	Limits should be $\frac{\pi}{4}$ and 0. Limits must be in radians.
	Obtain final answer $\frac{1}{2}\pi - \ln 2$	**A1**	OE exact answer. Need values for trig. functions and a single log term.
		5	

Question	Answer	Marks	Guidance
5	Square $a + \mathrm{i}b$, use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to 10 and $-4\sqrt{6}$ respectively	**M1**	
	Obtain $a^2 - b^2 = 10$ and $2ab = -4\sqrt{6}$	**A1**	Allow $2ab\mathrm{i} = -4\sqrt{6}\mathrm{i}$
	Eliminate one unknown and find an equation in the other	**M1**	Must be sensible algebra e.g. use of $\sqrt{a^2 - b^2} = a - b$ socres M0
	Obtain $a^4 - 10a^2 - 24[=0]$, or $b^4 + 10b^2 - 24[=0]$, or 3-term equivalent	**A1**	Or equivalent horizontal equation from correct work
	Obtain final answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	**A1**	e.g. $\pm\left(\sqrt{12} - \sqrt{2}\mathrm{i}\right)$ from correct work
	Alternative method for Question 5		
	Use the correct method to find the modulus and argument of $\sqrt{u}$	**M1**	
	Obtain modulus $\sqrt{14}$	**A1**	
	Obtain argument $\tan^{-1}\dfrac{-1}{\sqrt{6}}$ using an exact method	**A1**	e.g. by using half angle formula which gives $2\sqrt{6}t^2 - 10t - 2\sqrt{6} = 0$
	Convert to the required form	**M1**	$\pm\sqrt{14}\left(\dfrac{\sqrt{6}}{\sqrt{7}} - \dfrac{1}{\sqrt{7}}\mathrm{i}\right)$ This mark is available if working in decimals
	Obtain answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	**A1**	e.g. $\pm\left(\sqrt{12} - \sqrt{2}\mathrm{i}\right)$
		5	

Question	Answer	Marks	Guidance
6(a)	Express the LHS in terms of $\cos 2\theta$ and $\sin 2\theta$	**B1**	e.g. $\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**	e.g. $\frac{1-\left(1-2\sin^2\theta\right)}{2\sin\theta\cos\theta}$
	Obtain $\tan\theta$ from correct working	**A1**	AG
	Alternative method for Question 6(a)		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	**B1**	
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**	e.g. $\frac{1}{2\sin\theta\cos\theta} - \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{2\frac{\sin\theta}{\cos\theta}}\left(=\frac{4\sin^2\theta}{4\sin\theta\cos\theta}\right)$
	Obtain $\tan\theta$ from correct working	**A1**	AG
	Alternative method for Question 6(a)		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	**B1**	
	Use correct t substitution or rearrangement of $\sin 2\theta$ in terms of $\sec^2 2\theta$ and $\tan\theta$ to express the LHS in terms of $\tan\theta$.	**M1**	$\left(\frac{\sec^2\theta}{2\tan\theta} - \frac{1-\tan^2\theta}{2\tan\theta} = \right)\frac{1+\tan^2}{2\tan} - \frac{1-\tan^2}{2\tan}$
	Obtain $\tan\theta$ from correct working	**A1**	AG
		3	

Question	Answer	Marks	Guidance
6(b)	State integral of the form $\mp \ln\cos\theta$ or $\pm\ln\sec\theta$	*M1	$\left[-\ln\cos\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ OE
	Use correct limits correctly and insert exact values for the trigonometric ratios	DM1	Need to see evidence of the substitution
	Obtain a correct expression, e.g. $-\ln\frac{1}{2}+\ln\frac{1}{\sqrt{2}}$	A1	
	Obtain $\frac{1}{2}\ln 2$ from correct working	A1	AG (must see an intermediate step)
		4	

Question	Answer	Marks	Guidance
7	State equation $\frac{dy}{dx} = k\frac{y}{\sqrt{x+1}}$	**B1**	OE. Must be a differential equation.
	Separate variables correctly for *their* differential equation and integrate at least one side	***M1**	$\int \frac{1}{y} dy = \int \frac{k}{\sqrt{x+1}} dx$
	Obtain ln y	**A1**	Allow M1A1A1 if they have assumed $k = 1$ or are working with an incorrect value for k
	Obtain $2[k]\sqrt{x+1}$	**A1**	
	Use (0, 1) **and** (3, e) in an expression containing $\ln y$ and $\sqrt{x+1}$ and a constant of integration to determine k and/or a constant of integration c (or use (0, 1), (3, e) and (x, y) as limits on definite integrals)	**DM1**	If remove logs before finding the constant of integration then the constant must be of the correct form.
	Obtain $k = \frac{1}{2}$ and $c = -1$	**A1**	OE. $\left(\ln y = \sqrt{x+1} - 1\right)$ Their value of c will depend on where c is in their equation and whether they are working with $\frac{1}{k}\ln y$. The value of k must be consistent with what they integrated.
	Obtain $y = \exp\left(\sqrt{x+1} - 1\right)$	**A1**	NFWW, OE, ISW.
		7	

Question	Answer	Marks	Guidance
8	Use correct product (or quotient) rule	**M1**	At least 3 of 4 terms correct
	Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x\sec^2 x$	**A1**	OE.
	Equate *their* derivative to zero, use $\sec^2 x = 1 + \tan^2 x$ and obtain an equation in $\tan x$	**M1**	
	Obtain $2\tan^2 x - 5\tan x + 2 = 0$	**A1**	Allow $2\tan^3 x - 5\tan^2 x + 2\tan x = 0$
	State answer $x = 0$	**B1**	From correct derivative.
	Solve a 3 term quadratic in $\tan x$ and obtain a value of x	**M1**	Must be in radians
	Obtain answer, e.g. 0.464	**A1**	Must be 3 d.p. as specified in the question. Allow A1A0 if both values given to 2 d.p. or > 3 d.p.
	Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**	
	Alternative method for Question 8		
	Use correct product (or quotient) rule	**M1**	At least 3 of 4 terms correct
	Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x\sec^2 x$	**A1**	OE
	Equate *their* derivative to zero and obtain an equation in $\sin x$ and $\cos x$	**M1**	
	Obtain $5\cos x\sin x = 2$	**A1**	Or simplified equivalent (i.e. cancelled)
	State answer $x = 0$	**B1**	From correct derivative.
	Use double angle formula or square both sides and solve for x	**M1**	Or equivalent method. Must be in radians.
	Obtain answer, e.g. 0.464	**A1**	Must be 3 d.p. as specified in the question. Allow A1A0 if both values given to 2 d.p. or > 3 d.p.
	Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**	
		8	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{2+x} + \dfrac{B+Cx}{3+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	SOI
	Obtain one of $A = 4$, $B = 1$ and $C = -2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	ISW
		5	
9(b)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(3+x^2)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	**M1**	Allow unsimplified but not if still including ${}^nC_r$
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1 FT** **A1 FT**	$2\left(1-\frac{1}{2}x+\left(\frac{1}{2}x\right)^2 ....\right)$ $+\frac{1}{3}(1-2x)\left(1-\frac{1}{3}x^2....\right)$ The FT is on *their* A, B and C
	Multiply out, up to the terms in x^2, by $B + Cx$, where $BC \neq 0$	**M1**	Allow with B and C as implied in part **(b)**
	Obtain final answer $\frac{7}{3}-\frac{5}{3}x+\frac{7}{18}x^2$	**A1**	Or equivalent in form $p+qx+rx^2$. A0 if they multiply through by 18.
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply $CD = 2r - 2r\cos x$	B1	
	Using correct formulae for area of sector and trapezium, or equivalent, form an equation in r and x	M1	e.g. $2\times\frac{1}{2}r^2x = \frac{0.9}{2}(2r + 2r - 2r\cos x)r\sin x$
	Obtain $x = 0.9(2-\cos x)\sin x$	A1	AG, NFWW
		3	
10(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$	M1	Calculated for both values and correct for one value is sufficient for M1. Must be working in radians.
	Complete the argument correctly with correct values	A1	Must have sufficient accuracy to support the answer e.g. $\begin{matrix}0.5 > 0.484\\0.7 < 0.716\end{matrix}$ or $\begin{matrix}0.016 > 0\\-0.016 < 0\end{matrix}$ or $\begin{matrix}0.96... < 1\\1.02... > 1\end{matrix}$
		2	
10(c)	State a suitable equation, e.g. $\cos x = \left(2 - \frac{x}{0.9\sin x}\right)$	B1	If working in reverse, the first B1 is for $\frac{x}{0.9\sin x} = 2 - \cos x$
	Rearrange this as $x = 0.9\sin x(2-\cos x)$	B1	Need to see the complete sequence of changes.
		2	

Question	Answer	Marks	Guidance
10(d)	Use the iterative process correctly at least once	**M1**	Must be working in radians
	Obtain answer 0.62	**A1**	
	Show sufficient iterations to at least 4 d.p.to justify 0.62 to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	**A1**	Allow recovery. N.B. A candidate who starts with 0.5 and stops at 0.61 or starts at 0.7 and stops at 0.63 can score M1A0A1 if they have worked to the required accuracy.
		3	
11(a)	Show that $OA = OB = \sqrt{5}$	**B1**	CWO
	Evaluate the scalar product of the correct position vectors	**M1**	e.g. $(0-1+0)$ Condone of using *AO* and/or *BO*
	Divide *their* scalar product by the product of the moduli of *their* vectors and evaluate the inverse cosine of the result	**M1**	Much reach an angle. The question asks for the use of scalar product, so alternative methods (e.g. cosine rule) are not accepted.
	Obtain answer 101.5°	**A1**	The question asks for an answer in degrees. Accept 102° or better. Mark radians (1.77) as a misread. Do not ISW: 78.5° as final answer scores A0.
		4	

Question	Answer	Marks	Guidance
11(b)	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	Taking a general point of OM to have position vector $\lambda\mathbf{i} - \lambda\mathbf{k}$, express $AP = \sqrt{7}\,OA$ as an equation in λ	***M1**	$\lambda\left(\textit{their}\ \overline{OM}\right)$
	State a correct equation in any form	**A1**	e.g. $\sqrt{(-2+\lambda)^2 + 1 + (-\lambda)^2} = \sqrt{7}\sqrt{5}$
	Reduce to $\lambda^2 - 2\lambda - 15 = 0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
	Alternative method for Question 11(b)		
	State or imply that $OP = \gamma\sqrt{2}$	**B1**	
	State or imply that $\cos\frac{1}{2}AOB = \sqrt{\frac{2}{5}}$ and use cosine rule to form an equation in γ	***M1**	Allow $\cos\frac{1}{2}AOB = 0.632...$
	State a correct equation in any form	**A1**	e.g. $35 = 5 + 2\gamma^2 - 2\sqrt{5}.\gamma\sqrt{2}.\frac{\sqrt{2}}{\sqrt{5}}$
	Reduce to $\gamma^2 - 2\gamma - 15 = 0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates

Question	Answer	Marks	Guidance
11(b)	**Alternative method for Question 11(b)**		
	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	State or imply that $AM = \sqrt{3}$	**B1**	
	Use Pythagoras to find MP	***M1**	$MP = \sqrt{35 - (AM)^2}$
	Obtain $MP = 4\sqrt{2}$	**A1**	
	Correct method to find a position vector	**DM1**	$(\mathbf{i} - \mathbf{k}) \pm 4(\mathbf{i} - \mathbf{k})$
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
		6	

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State correct first two terms $1 + 2x$	**B1**	
	State a correct unsimplified version of the x^2 or x^3 term	**M1**	Symbolic binomial coefficients are not sufficient for the M mark.
	Obtain the next term $-x^2$	**A1**	
	Obtain the final term $\frac{4}{3}x^3$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	State or imply $u^2 - 3u - 1 = 0$, or equivalent in 4^x	**B1**	
	Solve for u or 4^x	**M1**	
	Obtain root $\frac{1}{2}\left(3+\sqrt{13}\right)$, or decimal in [3.30, 3.31]	**A1**	
	Use correct method for finding x from a positive root	**M1**	
	Obtain answer $x = 0.862$ and no other	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	State $\frac{dx}{dt} = 1 + \frac{1}{t+2}$	**B1**	
	Use product rule	**M1**	
	Obtain $\frac{dy}{dt} = e^{-2t} - 2(t-1)e^{-2t}$	**A1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain correct answer in any simplified form, e.g. $\frac{(3-2t)(t+2)}{t+3}\ e^{-2t}$	**A1**	
		5	
3(b)	Equate derivative to zero and solve for t	**M1**	
	Obtain $t = \frac{3}{2}$ and obtain answer $y = \frac{1}{2}e^{-3}$, or exact equivalent	**A1**	
		2	

Question	Answer	Marks	Guidance
4(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{4-x}$ and use a correct method to find a constant	**M1**	
	Obtain one of $A = 4$ and $B = -1$	**A1**	
	Obtain the second value	**A1**	
		3	
4(b)	Integrate and obtain terms $2\ln(1+2x)+\ln(4-x)$	**B1FT +B1FT**	The FT is on *A* and *B*.
	Substitute limits correctly in an integral of the form $a\ln(1+2x)+b\ln(4-x)$, where $ab \neq 0$	**M1**	
	Obtain final answer $\ln\left(\frac{50}{27}\right)$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	Use double angle formula to express $\tan 4\theta$ in terms of $\tan 2\theta$	**M1**	
	Use double angle formula to express result in terms of $\tan\theta$	**M1**	
	Obtain a correct equation in $\tan\theta$ in any form	**A1**	
	Obtain the given answer	**A1**	
		4	

Question	Answer	Marks	Guidance
5(b)	Solve for tan θ and obtain a value of θ	M1	
	Obtain answer, e.g. $53.5°$	A1	
	Obtain second answer, e.g. $126.5°$ and no other in the interval	A1	Ignore answers outside the given interval. Treat answers in radians as a misread.
		3	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = \cot\frac{1}{2}x$	B1	
	Sketch a second relevant graph, e.g. $y = 1 + e^{-x}$, and justify the given statement	B1	
		2	
6(b)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
6(c)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.34	A1	
	Show sufficient iterations to 4 d.p. to justify 1.34 to 2 d.p. or show there is a sign change in the interval (1.335, 1.345)	A1	
		3	

Question	Answer	Marks	Guidance
7(a)(i)	Justify the given statement $\dfrac{MN}{y} = \dfrac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	
		1	
7(a)(ii)	Express the area of *PMN* in terms of y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and equate to $\tan x$	**M1**	
	Obtain the given equation correctly	**A1**	
		2	
7(b)	Separate variables and integrate at least one side	**M1**	
	Obtain term $\dfrac{1}{6}y^3$	**A1**	
	Obtain term of the form $\pm\ln\cos x$	**M1**	
	Evaluate a constant or use $x = 0$ and $y = 1$ in a solution containing terms ay^3 and $\pm\ln\cos x$, or equivalent	**M1**	
	Obtain correct answer in any form, e.g. $\dfrac{1}{6}y^3 = -\ln\cos x + \dfrac{1}{6}$	**A1**	
	Obtain final answer $y = \sqrt[3]{(1 - 6\ln\cos x)}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
8(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \sqrt[4]{\mathrm{e}}$ and $y = \dfrac{1}{4\mathrm{e}}$, or exact equivalents	**A1**	
		4	
8(b)	Commence integration and reach $ax^{-3}\ln x + b\int x^{-3}.\ \dfrac{1}{x}\ \mathrm{d}x$	***M1**	
	Obtain $-\dfrac{1}{3}x^{-3}\ln x + \dfrac{1}{3}\int x^{-3}.\ \dfrac{1}{x}\ \mathrm{d}x$	**A1**	OE
	Complete integration and obtain $-\dfrac{1}{3}x^{-3}\ln x - \dfrac{1}{9}x^{-3}$	**A1**	
	Substitute limits correctly, having integrated twice	**DM1**	
	Obtain answer $\dfrac{1}{9} - \dfrac{1}{3}a^{-3}\ln a - \dfrac{1}{9}a^{-3}$	**A1**	OE
	Justify the given statement	**A1**	
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**B1**	OE
	Carry out a correct method to find $\overrightarrow{OD}$	**M1**	
	Obtain answer $-4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$	**A1**	OE
		3	
9(b)	State $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**B1FT**	OE. The FT is on $\overrightarrow{AB}$.
		1	
9(c)	For a general point *P* on *AB*, state $\overrightarrow{CP}$ or $\overrightarrow{DP}$ in component form, e.g. $\overrightarrow{CP} = (3-2\lambda, -\lambda, -6+2\lambda)$	***M1**	
	Equate a relevant scalar product to zero *or* equate derivative of $\lvert\overrightarrow{CP}\rvert$ to zero *or* use Pythagoras in a relevant triangle and solve for λ	**DM1**	
	Obtain $\lambda = 2$	**A1**	
	Show the perpendicular is of length 3	**A1**	
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**	
	Alternative method for Question 9(c)		
	Use a scalar product to find the projection *CN* (or *DN*) of *BC* (or *AD*) on *CD*	***M1**	
	Obtain *CN* = 3 (or *DN* = 3)	**A1**	
	Use Pythagoras to obtain *BN* (or *AN*)	**DM1**	

Question	Answer	Marks	Guidance
9(c) cont'd	Obtain answer 3	**A1**	
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	Substitute $-1+\sqrt{2}\mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**	
	Use $\mathrm{i}^2=-1$ at least once	**M1**	
	Complete the verification correctly	**A1**	
		3	
10(b)	State second root $-1-\sqrt{2}\mathrm{i}$	**A1**	
	Carry out a method to find a quadratic factor with zeros $-1\pm\sqrt{2}\mathrm{i}$	**M1**	
	Obtain z^2+2z+3	**A1**	
	Commence division and reach partial quotient z^2+kz	**M1**	
	Obtain second quadratic factor z^2-2z+4	**A1**	
	Solve a 3-term quadratic and use $\mathrm{i}^2=-1$	**M1**	
	Obtain roots $1+\sqrt{3}\mathrm{i}$ and $1-\sqrt{3}\mathrm{i}$	**A1**	
		7	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	B1	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	M1	Find point of intersection with $y = 2\|x-3\|$ or solve $2 - 5x$ with $2(x-3)$ or $-2(x-3)$
	Obtain $x = -\frac{4}{3}$	A1	
	State final answer $x < -\frac{4}{3}$	A1	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	B1	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	M1	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	A1	
	State final answer $x < -\frac{4}{3}$	A1	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sin 2\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 2 + 2\cos 2\theta$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$	**M1**	
	Obtain correct answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$	**A1**	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	**M1**	
	$\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	**M1 A1**	
	Simplify to given answer correctly $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$	**A1**	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set $= 2\theta$. State $\dfrac{dx}{dt} = \sin t$ or $\dfrac{dy}{dt} = 1 + \cos t$	**B1**	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**	
	Obtain correct answer $\dfrac{dy}{dx} = \dfrac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\dfrac{dy}{dx} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2=-1$ correctly at least once	M1	$1-5$ or $4+10$ seen
	Complete the verification correctly	A1	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	M1	
	Obtain x^2+2x+6	A1	
	Obtain root $x=\dfrac{3}{2}$	A1	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a)=2x^3+x^2+6x-18$	M1	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a=-18$	A1	
	$6a=-18$ $a=-3$ leading to $x=\dfrac{3}{2}$	A1	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\mathrm{i}\right)\left(-1+\sqrt{5}\mathrm{i}\right)a=9$	**M1 A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\mathrm{i}\right)+\left(-1+\sqrt{5}\mathrm{i}\right)+a=-\frac{1}{2}$	**M1 A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\frac{A}{1-x}+\frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A = 1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B = 1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C = 6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	A1 FT + A1 FT + A1 FT	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on *A*, *B*, *C*] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	A1	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D = -3$, $E = 4$ The FT is on *A*, *D*, *E*.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	*M1	$\frac{dy}{dx} = \left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = 4$	A1	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x} + b\int e^{-\frac{1}{2}x}\,dx$	*M1	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$	A1	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx} = 2e^{-\frac{1}{2}x} - xe^{-\frac{1}{2}x}$	*M1 A1	
	$\therefore 2xe^{-\frac{1}{2}x}$	A1	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\dfrac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form (12, – 4, 4)
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) . (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in *a* in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for *a*	**DM1**	Solve 3-term quadratic for *a* having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2 - \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert.\lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State that $1+e^{-3x}=e^2$	B1	With no errors seen to that point
	Use correct method to solve an equation of the form $e^{-3x}=a$, where $a>0$, for x or equivalent	M1	$\left(e^{-3x}=6.389...\right)$ Evidence of method must be seen.
	Obtain answer $x = -0.618$ only	A1	Must be 3 decimal places
	Alternative method for question 1		
	State that $1+e^{-3x}=e^2$	B1	
	Rearrange to obtain an expression for e^x and solve an equation of the form $e^x=a$, where $a>0$, or equivalent	M1	$e^x=\sqrt[3]{\dfrac{1}{e^2-1}}$
	Obtain answer $x = -0.618$ only	A1	Must be 3 decimal places
		3	

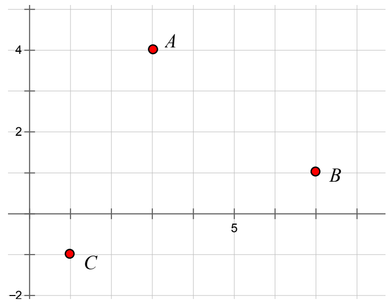
Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x or x^2 or x^3 term	M1	For the given expression
	State correct first two terms $1 + 2x$	A1	
	Obtain the next two terms $-4x^2+\dfrac{40}{3}x^3$	A1 + A1	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
2(b)	State answer $\lvert x\rvert<\dfrac{1}{6}$	B1	OE. Strict inequality
		1	

Question	Answer	Marks	Guidance
3(a)	State or imply $y\log 2 = \log 3 - 2x\log 3$	**B1**	Accept $y\ln 2 = (1-2x)\ln 3$
	State that the graph of y against x has an equation which is linear in x and y, or is of the form $ay = bx + c$	**B1**	Correct equation. Need a clear statement/comparison with matching linear form.
	Clear indication that the gradient is $-\dfrac{2\ln 3}{\ln 2}$	**B1**	Must be exact. Any equivalent e.g. $-\dfrac{2\log_k 3}{\log_k 2}$, $\log_2 \dfrac{1}{9}$
		3	
3(b)	Substitute $y = 3x$ in an equation involving logarithms and solve for x	**M1**	
	Obtain answer $x = \dfrac{\ln 3}{\ln 72}$	**A1**	Allow M1A1 for the correct answer following decimals
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$	M1	e.g. $\frac{\tan\theta+\tan 60°}{1-\tan\theta\tan 60°}=\frac{2}{\tan\theta}$
	Use $\tan 60°=\sqrt{3}$ and obtain a correct horizontal equation in any form	A1	e.g. $\tan\theta\left(\tan\theta+\sqrt{3}\right)=2\left(1-\sqrt{3}\tan\theta\right)$
	Reduce to $\tan^2\theta+3\sqrt{3}\tan\theta-2=0$ correctly	A1	AG
		3	
4(b)	Solve the given quadratic to obtain a value for θ	M1	$\left(\tan\theta=\frac{-3\sqrt{3}\pm\sqrt{35}}{2}=0.3599,-5.556\right)$
	Obtain one correct answer e.g. $\theta = 19.8°$	A1	Accept 1d.p. or better. If over-specified must be correct. 19.797...., 100.2029...
	Obtain second correct answer $\theta=100.2°$ and no others in the given interval	A1	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
5(a)	State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$	**B1**	CWO, AEF.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	**M1**	$y = \frac{1}{1+x^2}$
	$\frac{dy}{dx} = \frac{-2x}{\left(1+x^2\right)^2}$	**A1**	OE
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
		3	

Question	Answer	Marks	Guidance
5(b)	Use correct product rule to obtain $\frac{d}{d\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	**M1**	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	**A1**	e.g. $\pm\left(-2\cos^4\theta + 6\sin^2\theta\cos^2\theta\right)$
	Equate derivative to zero and obtain an equation in one trig ratio	**A1**	e.g. $3\tan^2\theta = 1$, or $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**	Or $-\frac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\frac{d^2y}{dx^2}$	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{-2\left(1+x^2\right)^2 + 2\times 2x\times 2x\left(1+x^2\right)}{\left(1+x^2\right)^4}$
	Equate derivative to zero and obtain an equation in x^2	**A1**	e.g. $6x^2 = 2$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**	
		4	

Question	Answer	Marks	Guidance
6(a)	Multiply numerator and denominator by 1 + i, or equivalent	**M1**	Must multiply out
	Obtain numerator 6 + 8i or denominator 2	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
	Alternative method for question 6(a)		
	Multiply out $(1-i)(x+iy)=7+i$ and compare real and imaginary parts	**M1**	
	Obtain $x+y=7$ or $y-x=1$	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
		3	
6(b)	Show the point *A* representing *u* in a relatively correct position	**B1 FT**	The FT is on $xy \neq 0$.
	Show the other two points *B* and *C* in relatively correct positions: approximately equal distance above / below real axis	**B1**	 Take the position of *A* as a guide to 'scale' if axes not marked
		2	

Question	Answer	Marks	Guidance
6(c)	State or imply arg(1 – i) = $-\frac{1}{4}\pi$	**B1**	Arg$C$
	Substitute exact arguments in arg(7 + i) – arg(1 – i) = arg u	**M1**	Must see a statement about the relationship between the Args e.g. Arg$A$ = Arg$B$ − Arg$C$ or equivalent exact method
	Obtain $\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi$ correctly	**A1**	Obtain given answer correctly from *their* $u = k(3+4i)$
		3	

Question	Answer	Marks	Guidance
7(a)	Correct separation of variables	**B1**	$\int \sec^2 2x\,\mathrm{d}x = \int \mathrm{e}^{-3t}\mathrm{d}t$ Needs correct structure
	Obtain term $-\frac{1}{3}\mathrm{e}^{-3t}$	**B1**	
	Obtain term of the form $k\tan 2x$	**M1**	From correct working
	Obtain term $\frac{1}{2}\tan 2x$	**A1**	
	Use $x = 0$, $t = 0$ to evaluate a constant, or as limits in a solution containing terms of the form $a\tan 2x$ and $b\mathrm{e}^{-3t}$, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	e.g. $\frac{1}{2}\tan 2x = -\frac{1}{3}\mathrm{e}^{-3t} + \frac{1}{3}$
	Obtain final answer $x = \frac{1}{2}\tan^{-1}\left(\frac{2}{3}\left(1-\mathrm{e}^{-3t}\right)\right)$	**A1**	
		7	
7(b)	State that x approaches $\frac{1}{2}\tan^{-1}\left(\frac{2}{3}\right)$	**B1 FT**	Correct value. Accept $x \to 0.294$ The FT is dependent on letting $\mathrm{e}^{-3t} \to 0$ in a solution containing e^{-3t}.
		1	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$	**B1**	Or equivalent seen or implied
	Use the correct process for calculating the modulus of both vectors to obtain *AB* and *CD*	**M1**	$AB = \sqrt{24}, CD = \sqrt{6}$
	Using exact values, verify that $AB = 2CD$	**A1**	Obtain given statement from correct work Allow from $BA = 2DC$, OE
		3	
8(b)	Use the correct process to calculate the scalar product of the relevant vectors (*their* $\overrightarrow{AB}$ and $\overrightarrow{CD}$)	**M1**	$\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$
	Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 99.6° (or 1.74 radians) or better	**A1**	Do not ISW if go on to subtract from 180° (99.594…, 1.738...) Accept 260.4°
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>8(c)</td><td>State correct vector equations for AB and CD in any form,
e.g. $(\mathbf{r} =)\begin{pmatrix}2\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r} =)\begin{pmatrix}1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td>B1ft</td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$
Alternative: $(\mathbf{r} =)\begin{pmatrix}4\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r} =)\begin{pmatrix}3\\2\\3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td></tr>
<tr><td></td><td>Equate at least two pairs of components of their lines and solve for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct pair of values from correct equations</td><td>A1</td><td>Alternatives when taking A or B as point on line
<table>
<tr><th>A</th><th>λ</th><th>μ</th><th></th><th>B</th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td></tr>
<tr><td>ik</td><td>$\frac{1}{2}$</td><td>1</td><td>$0\neq 2$</td><td>ik</td><td>$-\frac{1}{2}$</td><td>0</td><td>$0\neq 2$</td></tr>
<tr><td>jk</td><td>$\frac{3}{2}$</td><td>-3</td><td>$5\neq -5$</td><td>jk</td><td>$\frac{1}{2}$</td><td>-4</td><td>$5\neq -5$</td></tr>
</table></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td>A1</td><td>CWO with conclusion
e.g. $\frac{17}{3}\neq\frac{7}{3}$ or $\frac{17}{3}=\frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+4}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 3$, $B = -1$, $C = 3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
9(b)	Integrate and obtain $\ln(3x+2)\ldots$	**B1 FT**	The FT is on *A*
	State a term of the form $k\ln\left(x^2+4\right)$.	**M1**	From $\int \dfrac{\lambda x}{x^2+4}\mathrm{d}x$
	$\ldots-\dfrac{1}{2}\ln\left(x^2+4\right)\ldots$	**A1 FT**	The FT is on *B*
	$\ldots+\dfrac{3}{2}\tan^{-1}\dfrac{x}{2}$	**B1 FT**	The FT is on *C*
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln\left(x^2+4\right)$ and $c\tan^{-1}\left(\dfrac{x}{2}\right)$, and subtract in correct order	**M1**	Using terms that have been obtained correctly from completed integrals
	Obtain answer $\dfrac{3}{2}\ln 2+\dfrac{3}{8}\pi$, or exact 2-term equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a=\frac{1}{2a}$	**A1**	Obtain given answer from correct working**.** The question says 'show that ..' so there should be an intermediate step e.g. $\cos x=2x\sin x$. Allow $\tan x=\frac{1}{2x}$
		3	
10(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	**M1**	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	**A1**	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	**A1**	$3, 3.3067, 3.2917, 3.2923$ Allow more than 4d.p. Condone truncation.
		3	

Question	Answer	Marks	Guidance
10(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2 \mathrm{d}x$	B1	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos 2A formula, commence integration by parts and reach $x(ax+b\sin 2x)\pm\int ax+b\sin 2x\mathrm{d}x$	*M1	Alternative: $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\int\frac{1}{4}\sin 2x\mathrm{d}x$
	Obtain $x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int\frac{1}{2}x+\frac{1}{4}\sin 2x\,\mathrm{d}x$, or equivalent	A1	
	Complete integration and obtain $\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x$	A1	OE
	Substitute limits $x = 0$ and $x = \frac{1}{2}\pi$, having integrated twice	DM1	$\frac{\pi}{2}\left[\frac{\pi^2}{8}+0-\frac{1}{4}-0-0-\frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2-4\right)$, or exact equivalent	A1	CAO
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\|x-3\|$ or solve $2 - 5x$ with $2(x-3)$ or $-2(x-3)$
	Obtain $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2-5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2+2\cos 2\theta$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	**M1**	
	$\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$	**B1**	
	$\frac{dy}{dx} = \frac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	**M1 A1**	
	Simplify to given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set $= 2\theta$. State $\frac{dx}{dt} = \sin t$ or $\frac{dy}{dt} = 1 + \cos t$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	B1	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	B1	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	M1	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	A1	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to 301.8°	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to 2.9(2)° and no others in the interval	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2 = -1$ correctly at least once	M1	$1-5$ or $4+10$ seen
	Complete the verification correctly	A1	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	M1	
	Obtain $x^2 + 2x + 6$	A1	
	Obtain root $x=\dfrac{3}{2}$	A1	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a) = 2x^3 + x^2 + 6x - 18$	M1	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a = -18$	A1	
	$6a = -18$ $a = -3$ leading to $x=\dfrac{3}{2}$	A1	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\text{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\text{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\text{i}\right)\left(-1+\sqrt{5}\text{i}\right)a = 9$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\text{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\text{i}\right)+\left(-1+\sqrt{5}\text{i}\right)+a = -\frac{1}{2}$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\frac{A}{1-x}+\frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A = 1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B = 1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C = 6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	A1 FT + A1 FT + A1 FT	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on *A*, *B*, *C*] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	A1	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D = -3$, $E = 4$ The FT is on *A*, *D*, *E*.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	***M1**	$\frac{dy}{dx} = \left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**DM1**	
	Obtain $x = 4$	**A1**	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}dx$	*M1	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}dx$	A1	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	*M1 A1	
	$\therefore 2xe^{-\frac{1}{2}x}$	A1	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\dfrac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form (12, – 4, 4)
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) \cdot (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert.\lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
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GENERIC MARKING PRINCIPLE 6:
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular equation $4^2\left(5^x-1\right)^2=\left(5^x\right)^2$ or pair of equations $4\left(5^x-1\right)=\pm 5^x$	M1	
	Obtain $5^x=\frac{4}{3}$ and $5^x=\frac{4}{5}$ (or $5^{x+1}=4$)	A1	
	Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a > 0$, or $b > 0$	M1	
	Obtain answers $x = 0.179$ and $x = -0.139$	A1	
	Alternative method for question 1		
	Obtain $5^x=\frac{4}{3}$ by solving an equation	B1	
	Obtain $5^x=\frac{4}{5}$ (or $5^{x+1}=4$) by solving an equation	B1	
	Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a > 0$, or $b > 0$	M1	
	Obtain answers $x = 0.179$ and $x = -0.139$	A1	
		4	

Question	Answer	Marks	Guidance
2(a)	State $R = \sqrt{34}$	**B1**	
	Use trig formulae to find α	**M1**	$\tan\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{3}{\sqrt{34}}$ or $\cos\alpha = \frac{5}{\sqrt{34}}$.
	Obtain $\alpha = 0.54$	**A1**	$30.96°$ scores **M1A0**.
		3	
2(b)	State greatest value 34	**B1 FT**	*Their* R^2 .
	State least value 0	**B1**	
		2	

Question	Answer	Marks	Guidance
3(a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = e^{1-2x} - 2xe^{1-2x}$
	Equate derivative to zero and solve for x	M1	
	Obtain $x = \frac{1}{2}$ and $y = \frac{1}{2}$	A1	
		4	
3(b)	Use a correct method for determining the nature of a stationary point	M1	e.g. $\frac{d^2y}{dx^2} = -2e^{1-2x} - 2(1-2x)e^{1-2x}$
	Show that it is a maximum point	A1	
		2	

Question	Answer	Marks	Guidance
4	State that $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$ or $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$	**B1**	
	Substitute throughout for x and $\mathrm{d}x$	**M1**	
	Obtain a correct integral with integrand $\frac{2}{u^2+1}$	**A1**	
	Integrate and obtain term of the form $k\tan^{-1}u$	**M1**	$\left(2\tan^{-1}u\right)$
	Use limits $\sqrt{3}$ and ∞ for u or equivalent and evaluate trig.	**A1**	e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians.
	Obtain answer $\frac{1}{3}\pi$	**A1**	Or equivalent single term.
		6	

Question	Answer	Marks	Guidance
5(a)	Use correct trig formulae and express equation in terms of $\tan\theta$	**M1**	
	Obtain a correct equation in $\tan\theta$ in any form	**A1**	e.g. $\frac{1-\tan^2\theta}{2\tan\theta}+\frac{1}{\tan\theta}=2$
	Reduce to $\tan^2\theta+4\tan\theta-3=0$, or 3-term equivalent	**A1**	
		3	
5(b)	Solve a 3-term quadratic for tan θ and calculate θ	**M1**	$\left(\tan\theta=-2\pm\sqrt{7}\right)$
	Obtain answer, e.g. 0.573	**A1**	Must be 3 d.p.
	Obtain second answer, e.g. 1.783 and no other	**A1**	Ignore answers outside the given interval. Treat answers in degrees as a misread. $(32.9°, 102.1°)$
		3	

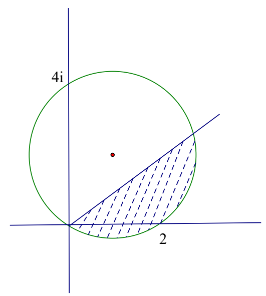
Question	Answer	Marks	Guidance
6	State or imply 1 + 2x as first terms of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	Form an expression for the coefficient of x or coefficient of x^2 in the expansion of $(a+bx)\sqrt{1+4x}$ and equate to given coefficient	**M1**	All relevant terms considered.
	Obtain $2a + b = 3$, or equivalent	**A1**	One correct equation.
	Obtain– $2a + 2b = -6$ or equivalent	**A1**	Second correct equation.
	Obtain answer $a = 2$ and $b = -1$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	Show sufficient working to justify the given answer	**B1**	
		1	
7(b)	Correct separation of variables	**B1**	e.g. $-\int \frac{1}{t}\mathrm{d}t = \int \frac{1}{x\ln x}\mathrm{d}x$
	Obtain term $\ln(\ln x)$	**B1**	
	Obtain term $-\ln t$	**B1**	
	Evaluate a constant or use $x = \mathrm{e}$ and $t = 2$ as limits in an expression involving $\ln(\ln x)$	**M1**	
	Obtain correct solution in any form, e.g. $\ln(\ln x) = -\ln t + \ln 2$	**A1**	
	Use log laws to enable removal of logarithms	**M1**	
	Obtain answer $x = \mathrm{e}^{\frac{2}{t}}$, or simplified equivalent	**A1**	
		7	
7(c)	State that x tends to 1 coming from $x = \mathrm{e}^{\frac{k}{t}}$	**B1**	
		1	

Question	Answer	Marks	Guidance
8(a)	Commence integration and reach $a\sqrt{x}\ln x + b\int\sqrt{x}\,.\,\frac{1}{x}$ dx, or equivalent	*M1	
	Obtain $2\sqrt{x}\ln x - \int 2\sqrt{x}\,.\,\frac{1}{x}$ dx, or equivalent	A1	
	Obtain integral $2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits and equate result to 6	DM1	
	Rearrange and obtain $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$	A1	Obtain **given answer** from full and correct working.
		5	
8(b)	Calculate the values of a relevant expression or pair of expressions at $a = 9$ and $a = 11$	M1	e.g. $\begin{cases} 9 < 10.31 \\ 11 > 9.99 \end{cases}$ or $1.31 > 0, -1.01 < 0$
	Complete the argument correctly with correct values	A1	
		2	
8(c)	Use the iterative process $a_{n+1} = \exp\left(\frac{1}{\sqrt{a_n}} + 2\right)$ correctly at least once	M1	
	Obtain answer 10.12	A1	
	Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125)	A1	e.g. $10, 10.1374, 10.1156, 10.1190, ...$ $9, 10.3123, 10.0886, 10.1233, 10.1178, ...$ $11, 9.9893, 10.1391, 10.1153, 10.1191, ...$
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct method to evaluate the scalar product of relevant vectors	**M1**	$(-4-2+6)$
	Obtain answer zero and deduce the given statement	**A1**	Need a conclusion or a statement in advance that the scalar product will be zero.
		2	
9(b)	Express general point of *l* or *m* in component form, e.g. $(3 + 4s, 2 - s, 5 + 3s)$ or $(1 - t, -1 + 2t, -2 + 2t)$	**B1**	
	Equate at least two pairs of components and solve for *s* or for *t*	**M1**	
	Obtain correct answer $s = -1$ and $t = 2$	**A1**	
	Verify that all three equations are satisfied	**A1**	
	State position vector of the intersection $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, or equivalent	**A1**	Can come from 1 correct value and no contradictory statement.
		5	

Question	Answer	Marks	Guidance
9(c)	Taking a general point P on m, form an equation in t by ***either*** equating a relevant scalar product to zero, ***or*** equating the derivative of $\left\|\overrightarrow{OP}\right\|$ to zero, ***or*** taking a specific point Q on m, e.g. (1, – 1, – 2), using Pythagoras in triangle OPQ	***M1**	e.g. $\begin{pmatrix} 1-t \\ -1+2t \\ -2+2t \end{pmatrix}.\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}=0$
	Obtain $t=\dfrac{7}{9}$	**A1**	
	Carry out correct method to find OP	**DM1**	
	Obtain $\dfrac{\sqrt{5}}{3}$	**A1**	Obtain the **given answer** from full and correct working.
	Alternative method for question 9(c)		
	Take a specific point Q on m, e.g. (– 1, 3, 2) and use a scalar product to find QN, the projection of OQ on m	***M1**	
	Obtain $QN=\dfrac{11}{3}$, or equivalent	**A1**	
	Use Pythagoras to obtain ON	**DM1**	
	Obtain the given answer correctly	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	Substitute 1 + 2i in the polynomial and attempt expansions of x^2 and x^3	**M1**	$u^2 = -3+4\text{i},\ u^3 = -11-2\text{i}$ Full substitution but need not simplify.
	Equate real and/or imaginary parts to zero	**M1**	$-18-3a+b=0,\ 4+4a=0$
	Obtain $a = -1$	**A1**	
	Obtain $b = 15$	**A1**	
		4	
10(b)	State second root 1 – 2i	**B1**	
		1	
10(c)	State the quadratic factor x^2-2x+5	**B1**	
	State the linear factor $2x+3$	**B1**	
		2	
10(d)(i)	Show a circle with centre 1 + 2i	**B1**	
	Show circle passing through the origin	**B1**	
	Show the half line $y = x$ in the first quadrant (accept chord of circle)	**B1**	
	Shade the correct region on a correct diagram	**B1**	
		4	
10(d)(ii)	State answer $2-\sqrt{5}$	**B1**	
		1	

Cambridge International A Level

MATHEMATICS	**9709/32**
Paper 3 Pure Mathematics 3	**October/November 2021**
MARK SCHEME	
Maximum Mark: 75	

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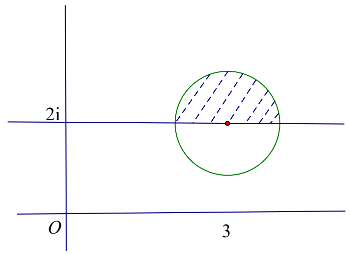
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product, a quotient or power	*M1	e.g. $\ln\left(7^x\right) = x\ln 7$
	Obtain a correct linear equation in any form	A1	e.g. $\ln 3 + (1-x)\ln 2 = x\ln 7$
	Solve a linear equation for x	DM1	
	Obtain answer $x = \dfrac{\ln 6}{\ln 14}$	A1	Maximum 3 out of 4 available if final answer not in required form e.g. 0.67… ISW once correct answer seen.
	Alternative method for Question 1		
	$2^{1-x} = 2\times 2^{-x}$	*M1	OE
	$6 = 2^x 7^x \left[= 14^x\right]$	A1	
	Use law of the logarithm of a power to solve for x	DM1	Must be a linear power. Allow $x = \ln_{14}(6)$.
	Obtain answer $x = \dfrac{\ln 6}{\ln 14}$	A1	ISW once correct answer seen.
		4	

Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(3x-a)^2 > 2^2(x+2a)^2$, or corresponding quadratic equation, or pair of linear equations or linear inequalities	**B1**	Need 2^2 seen or implied.
	Make reasonable attempt to solve a 3-term quadratic, or solve two linear equations for x in terms of a	**M1**	$\left(5x^2 - 22ax - 15a^2 = 0\right)$
	Obtain critical values $x = 5a$ and $x = -\frac{3}{5}a$ and no others	**A1**	OE Accept incorrect inequalities with correct critical values. Must state 2 values i.e. $\frac{a \pm b}{c}$ is not sufficient.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**A1**	Do not condone $\geqslant$ for > or $\leqslant$ for < in the final answer. $5a < x < -\frac{3}{5}a$ is **A0**, 'and' is **A0**.
	Alternative method for Question 2		
	Obtain critical value $x = 5a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{3}{5}a$ similarly	**B2**	Maximum 2 marks if more than 2 critical values.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**B1**	Do not condone $\geq$ for > or $\leq$ for < in the final answer. $5a < x < -\frac{3}{5}a$ is **B0**, 'and' is **B0**.
		4	

Question	Answer	Marks	Guidance
3(a)	Substitute for u and w and state correct conjugate of one side	**B1**	
	Express the other side without conjugates and confirm $(u+w)^* = u^* + w^*$	**B1**	**Given answer.** Needs explicit reference to conjugate of both sides.
		2	
3(b)	Substitute and remove conjugates to obtain a correct equation in x and y	**B1**	e.g. $x+2-(y+1)\mathrm{i}+(2+\mathrm{i})(x+\mathrm{i}y)=0$
	Use $\mathrm{i}^2=-1$ and equate real and imaginary parts to zero	**M1**	
	Obtain two correct equations in x and y	**A1**	e.g. $3x-y+2=0$ and $x+y-1=0$. Allow $x\mathrm{i}+y\mathrm{i}-\mathrm{i}=0$.
	Solve and obtain answer $z=-\frac{1}{4}+\frac{5}{4}\mathrm{i}$	**A1**	Allow for real and imaginary parts stated separately.
		4	

Question	Answer	Marks	Guidance
4	State or imply the form $A + \frac{B}{2x-1} + \frac{C}{x-3}$	**B1**	$\frac{Dx+E}{2x-1} + \frac{F}{x-3}$ and $\frac{P}{2x-1} + \frac{Qx+R}{x-3}$ are also valid.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -3$ and $C = 2$	**A1**	Allow maximum **M1A1** for one or more 'correct' values after **B0**.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	Alternative method for Question 4		
	Divide numerator by denominator	**M1**	
	Obtain $2\left[+\frac{Px+Q}{(2x-1)(x-3)}\right]$	**A1**	$\left[2+\frac{x+7}{(2x-1)(x-3)}\right]$
	State or imply the form $\frac{D}{2x-1} + \frac{E}{x-3}$	**B1**	
	Obtain one of $D = -3$ and $E = 2$	**A1**	
	Obtain a second value	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Show circle with centre 3 + 2i	**B1**	
	Show circle with radius 1. Must match *their* scales: if scales not identical should have an ellipse.	**B1**	
	Show line $y = 2$ in at least the diameter of a circle in the first quadrant	**B1**	
	Shade the correct region in a correct diagram	**B1**	
		4	
5(b)	Identify the correct point	**B1**	
	Carry out a correct method for finding the argument	**M1**	e.g. $\arg x = \tan^{-1}\frac{2}{3} + \sin^{-1}\frac{1}{\sqrt{13}}$ Exact working required.
	Obtain answer 49.8°	**A1**	Or better. 0.869 radians scores **B1M1A0**.
		3	**Special Case 1: B1M0** for 45° if they have shaded the wrong half of the circle. **Special Case 2:** 3 out of 3 available if they identify the correct point on the correct circle and it is consistent with *their* shading.

Question	Answer	Marks	Guidance
6(a)	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	**B1**	
	Substitute expansions in $\frac{1}{2}(\sin 5x + \sin x)$, or equivalent	**M1**	
	Simplify and obtain $\frac{1}{2}(\sin 5x + \sin x) = \sin 3x \cos 2x$	**A1**	Obtain the **given identity** correctly.
		3	
6(b)	Obtain integral $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x$, or equivalent	**B1**	
	Substitute limits correctly in an expression of the form $p\cos 5x + q\cos x$	**M1**	Correct limits and subtracted the right way around. Do not need values of trig functions for M1. Maximum one slip.
	Obtain $\frac{1}{5}\left(3-\sqrt{2}\right)$	**A1**	Substitute values and obtain the **given answer** following full, correct and exact working.
		3	

Question	Answer	Marks	Guidance
7	Separate variables correctly	**B1**	$\int \frac{1}{y^2}\,dy = \int 4xe^{-2x}\,dx$
	$\int \frac{1}{y^2}\,dy = -\frac{1}{y}$	**B1**	OE
	Commence the other integration and reach $axe^{-2x} + b\int e^{-2x}\,dx$	**M1**	
	Obtain $-2xe^{-2x} + 2\int e^{-2x}\,dx$ or $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$	**A1**	SOI (might have taken out factor of 4)
	Complete integration and obtain $-2xe^{-2x} - e^{-2x}$	**A1**	
	Evaluate a constant or use $x = 0$ and $y = 1$ as limits in a solution containing terms of the form $\frac{p}{y}$, qxe^{-2x}, re^{-2x}, or equivalent.	**M1**	
	Obtain $y = \frac{e^{2x}}{2x+1}$, or equivalent expression for y	**A1**	ISW
		7	

Question	Answer	Marks	Guidance
8(a)	Expand the square and equate to 1	**B1**	
	Use correct double angle formula	**M1**	Need to see $\frac{4}{2}$ or $\sin 2\theta = 2\sin\theta\cos\theta$ stated.
	Obtain $\cos^4\theta + \sin^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$	**A1**	Obtain the **given result** correctly.
		3	
8(b)	Use the identity and carry out a method for finding a root	**M1**	$\left(1 - \frac{1}{2}\sin^2 2\theta = \frac{5}{9}\right)$
	Obtain answer $35.3°$	**A1**	Must be correct if overspecified: 35.264...
	Obtain a second answer, e.g. $54.7°$	**A1 FT**	[e.g $90° -$ *their* $35.3°$] Do not FT if mixing degrees and radians.
	Obtain the remaining answers, e.g. $144.7°$ and $125.3°$ and no others in the given interval	**A1 FT**	[e.g. $180° - ..$ and $180° - ..$] Ignore answers outside the given interval. Treat answers in radians as a misread. (0.615, 0.955, 2.19, 2.53) Do not FT if mixing degrees and radians.
		4	

Question	Answer	Marks	Guidance
9(a)	State correct derivative of ye^{2x} with respect to x	B1	$2ye^{2x}+e^{2x}\frac{dy}{dx}$
	State correct derivative of y^2e^x with respect to x	B1	$2ye^x\frac{dy}{dx}+y^2e^x$
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	A1	Obtain the **given answer** correctly. Condone multiplication by $\frac{-1}{-1}$ and cancelling of e^x without comment.
	Alternative method for Question 9(a)		
	Rearrange as $y=\frac{2}{e^{2x}-ye^x}$ $\Rightarrow \frac{d}{dx}\left(e^{2x}-ye^x\right)=2e^{2x}-ye^x-e^x\frac{dy}{dx}$	B1	Other rearrangements are possible e.g. $y=2e^{-2x}+y^2e^{-x}$ $\frac{d}{dx}\left(y^2e^{-x}\right)=2ye^{-x}\frac{dy}{dx}-y^2e^{-x}$
	$\frac{dy}{dx}=-\frac{2}{\left(e^{2x}-ye^x\right)^2}\times\left(2e^{2x}-ye^x-e^x\frac{dy}{dx}\right)$	B1	$\Rightarrow\frac{dy}{dx}=-4e^{-x}+2ye^{-x}\frac{dy}{dx}-y^2e^{-x}$
	Solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	A1	Obtain the **given answer** correctly.
		4	

Question	Answer	Marks	Guidance
9(b)	Equate denominator to zero and substitute for y or for e^x in the equation of the curve	*M1	
	Obtain equation of the form $ae^{3x} = b$ or $cy^3 = d$	DM1	$(e^{3x} = 8, \quad y^3 = 1)$ SOI
	Obtain $x = \ln 2$	A1	Accept $\frac{1}{3}\ln 8$ ISW
	Obtain $y = 1$	A1	
		4	

Question	Answer	Marks	Guidance
10(a)	Obtain direction vector $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$, or equivalent	B1	Accept answers as column vectors throughout.
	Use a correct method to form a vector equation	M1	
	State answer $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, or equivalent correct form	A1	e.g. $\mathbf{r} = \begin{pmatrix}0\\3\\1\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ Allow $\begin{pmatrix}x\\y\\z\end{pmatrix}$ for $\mathbf{r}$.
		3	
10(b)	Use a correct method to find the position vector of C	M1	e.g. $\mathbf{OC} = \mathbf{OA} + \mathbf{AC} = \begin{pmatrix}1-3\\2+3\\-1+6\end{pmatrix}$
	Obtain answer $-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$, or equivalent	A1	Accept as coordinates.
		2	

Question	Answer	Marks	Guidance
10(c)	State $\overrightarrow{OP}$ in component form	**B1 FT**	
	Form an equation in λ by equating the modulus of OP to $\sqrt{14}$, or equivalent	**M1**	
	Simplify and obtain $3\lambda^2 - \lambda - 4 = 0$, or equivalent	**A1**	$3\lambda^2 + \lambda - 4 = 0$ if using $\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ in **(a)**. $3\mu^2 + 5\mu - 2 = 0$ if using $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ in **(a)** and OB.
	Solve a 3-term quadratic and find a position vector	**M1**	$\left(\lambda = -1, \frac{4}{3} \text{ or } \lambda = 1, -\frac{4}{3} \text{ or } \mu = \frac{1}{3}, -2 \text{ or } \mu = -\frac{1}{3}, 2\right)$
	Obtain answers $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i} + \frac{10}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$, or equivalent	**A1**	Accept as coordinates.
		5	

Question	Answer	Marks	Guidance
11(a)	Use chain rule	**M1**	Allow if not starting with the correct index.
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of tan x	**A1**	e.g. $\frac{dy}{dx} = \frac{1 + \tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	**B1**	Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$.
		4	

Question	Answer	Marks	Guidance
11(b)	Equate answer to part (**a**) to 1 and obtain a quartic equation in t or tan x	***M1**	At least as far as $\left(1+\tan^2 x\right)^2 = 4\tan x$.
	Obtain correct answer, i.e. $t^4 + 2t^2 - 4t + 1 = 0$	**A1**	Or equivalent horizontal form.
	Commence division by $t - 1$	**DM1**	As far as $t^3 + t^2 + \ldots$by long division or inspection. Allow verification by multiplying given answer by $t - 1$.
	Obtain the given answer	**A1**	
		4	
11(c)	Use the iterative process correctly with the given formula at least once	**M1**	Obtain one value and use that to obtain the next. Must be working in radians.
	Obtain final answer a = 0.29	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295)	**A1**	e.g. 0.3, 0.2854, 0.2894, 0.2883, ... 0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ... 0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ...
		3	

Cambridge Assessment International Education

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MATHEMATICS	**9709/33**
Paper 3 Pure Mathematics 3	**October/November 2021**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Commence division and reach partial quotient of the form $2x^2 + kx$	**M1**	
	Obtain quotient $2x^2 + 2x - 2$	**A1**	
	Obtain remainder $-6x + 5$	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	Show a recognizable sketch graph of $y = \|2x - 3\|$	**B1**	
		1	

Question	Answer	Marks	Guidance
2(b)	Find x-coordinate of intersection with $y = 3x + 2$	M1	
	Obtain $x = \frac{1}{5}$	A1	
	State final answer $x > \frac{1}{5}$ only	A1	
	Alternative method for Question 2(b)		
	Solve the linear inequality $3 - 2x < 3x + 2$, or corresponding equation	M1	
	Obtain critical value $x = \frac{1}{5}$	A1	
	State final answer $x > \frac{1}{5}$ only	A1	
	Alternative method for Question 2(b)		
	Solve the quadratic inequality $(2x-3)^2 < (3x+2)^2$, or corresponding equation	M1	
	Obtain critical value $x = \frac{1}{5}$	A1	
	State final answer $x > \frac{1}{5}$ only	A1	
		3	

Question	Answer	Marks	Guidance
3	Use laws of indices correctly and solve for 4^x	**M1**	
	Obtain correct solution in any form, e.g. $4^x = \frac{256}{15}$	**A1**	
	Use a correct method for solving an equation of the form $4^x = a$, where $a > 0$	**M1**	
	Obtain answer 2.047	**A1**	
		4	

Question	Answer	Marks	Guidance
4	Commence integration and reach $ax\cos\frac{1}{2}x + b\int\cos\frac{1}{2}x\,dx$	***M1**	
	Obtain $-2x\cos\frac{1}{2}x + 2\int\cos\frac{1}{2}x\,dx$	**A1**	OE
	Complete integration obtaining $-2x\cos\frac{1}{2}x + 4\sin\frac{1}{2}x$	**A1**	OE
	Use limits correctly, having integrated twice	**DM1**	
	Obtain answer $2 + \frac{\sqrt{3}}{3}\pi$, or exact equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
5	Use double angle formula and obtain an equation in $\sin\theta$	**M1**	
	Reduce to $6\sin^2\theta + \sin\theta - 5 = 0$, or 3-term equivalent	**A1**	
	Solve a 3-term quadratic in $\sin\theta$ and calculate θ	**M1**	
	Obtain answer, e.g. $56.4°$	**A1**	
	Obtain second and third answers, e.g. $123.6°$ and $270°$ and no others in the given interval	**A1**	Ignore answers outside the interval. Treat answers in radians as a misread.
		5	

Question	Answer	Marks	Guidance
6(a)	Use $\cos(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**	
	Collect terms and reach $2\cos x + \sqrt{3}\sin x$	**A1**	
	State $R = \sqrt{7}$	**A1**	
	Use trig formula to find α	**M1**	
	Obtain $\alpha = 40.89°$	**A1**	
		5	
6(b)	Use correct method to find x	**M1**	
	Obtain answer $x = 220.9°$	**A1**	
		2	

Question	Answer	Marks	Guidance
7(a)	Use chain rule to differentiate LHS	*M1	
	Obtain $\frac{1}{x+y}\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	A1	
	Equate derivative of LHS to $1-2\frac{\mathrm{d}y}{\mathrm{d}x}$ and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	DM1	
	Obtain the given answer correctly	A1	
		4	
7(b)	State $x + y = 1$	B1	
	Obtain or imply $x - 2y = 0$	B1	
	Obtain coordinates $x = \frac{2}{3}$ and $y = \frac{1}{3}$	B1	
		3	

Question	Answer	Marks	Guidance
8(a)	State $\overrightarrow{OM} = 4\mathbf{i} + 2\mathbf{j}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	
	Obtain answer $3\mathbf{j} + \mathbf{k}$	A1	
	Use a correct method to find a line equation for *MN*	M1	
	Obtain answer $\mathbf{r} = 3\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} - \mathbf{j} - \mathbf{k})$, or equivalent	A1	
		5	

Question	Answer	Marks	Guidance
8(b)	Taking a general point *P* on *MN*, form an equation in λ by *either* equating a relevant scalar product to zero *or* equating the derivative of $\overrightarrow{OP}$ to zero *or* using Pythagoras in triangle *OPM* or *OPN*	**M1**	
	Obtain $\lambda = \frac{2}{9}$	**A1**	OE
	Use correct method to find *OP*	**M1**	
	Obtain the given answer correctly	**A1**	
	Alternative method to Question 8(b)		
	Use a scalar product to find the projection of *OM* (or *ON*) on *MN*	**M1**	
	Obtain answer $\frac{14}{\sqrt{18}}\left(\text{or } \frac{4}{\sqrt{18}}\right)$	**A1**	
	Use Pythagoras to obtain the perpendicular	**M1**	
	Obtain the given answer correctly	**A1**	
		4	

Question	Answer	Marks	Guidance
9(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 3$	**A1**	
		4	
9(b)	State $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$, or $\mathrm{d}x = 2\sqrt{x}\mathrm{d}u$, or $2u\ \mathrm{d}u = \mathrm{d}x$	**B1**	
	Substitute and obtain integrand $\frac{2}{9-u^2}$	**B1**	
	Use given formula for the integral or integrate relevant partial fractions	**M1**	
	Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$	**A1**	OE
	Use limits $u = 0$ and $u = 2$ correctly	**M1**	
	Obtain the given answer correctly	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	State or imply equation of the form $\frac{dx}{dt} = k\frac{x}{20-x}$	**M1**	
	Obtain $k = 19$	**A1**	**AG**
		2	
10(b)	Separate variables and integrate at least one side	**M1**	
	Obtain terms $20 \ln x - x$ and $19t$, or equivalent	**A1 A1**	
	Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms $a \ln x$ and bt	**M1**	
	Substitute $t = 1$ and rearrange the equation in the given form	**A1**	**AG**
		5	
10(c)	Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once	**M1**	
	Obtain final answer $x = 2.83$	**A1**	
	Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835)	**A1**	
		3	
10(d)	Set $x = 20$ and obtain answer $t = 2.15$	**B1**	
		1	

Question	Answer	Marks	Guidance
11(a)	State or imply $r = 2$	**B1**	
	State or imply $\theta = \frac{5}{6}\pi$	**B1**	
		2	
11(b)	Use a correct method for finding the modulus or argument of u^6	**M1**	
	Show correctly that u^6 is real and has value -64	**A1**	
		2	
11(c)(i)	Show half lines from the point representing $-\sqrt{3} + \mathrm{i}$	**B1**	
	Show correct half lines	**B1**	
	Show the line $x = 2$ in the first quadrant	**B1**	
	Shade the correct region	**B1**	
		4	
11(c)(ii)	Carry out a correct method to find the greatest value of $\lvert z\rvert$	**M1**	
	Obtain answer 5.14	**A1**	
		2	